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A PRACTICAL TREATISE

ON THE CONSTRUCTION OF

OBLIQUE BRIDGES,

WITH SPIRAL AND WITH EQUILIBRATED COURSES.

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LONDON:
E. & F. N. SPON, 16, BUCKLERSBURY.

1855.

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PREFACE.

THE following Treatise on the Construction of Oblique Bridges is intended for the use of those who are engaged in the erection of such structures. The introduction of mathematical investigations was unavoidable, but these may be omitted by the reader who is content to be guided by rules without knowing the reasons for them.

Although grave objections may be urged against the use of *spiral courses* for oblique bridges, yet the accuracy of form that can be given to the arch-stones may render it advisable, under proper limitations, to adopt this in preference to a better arrangement of the courses, that does not admit of like exactness in the execution of the work. The methods of Part I. are substantially the same as those of Messrs. Nicholson and Buck, but there are numerous variations in the details.

Oblique bridges were first constructed with *equilibrated courses* by Mr. Adie, who communicated a paper on the subject to the Institution of Civil Engineers, in 1842. Dr. Whewell and Mr. Sang have also published some theoretical investigations on the subject. In Part II. I have endeavoured to supply information respecting this system in a form intelligible to the practical man.

The combination of the direct and oblique arches described in Part III. was also first used by Mr. Adie, excepting that his coursing-joints on the developments of the soffits were arcs of circles instead of the more appropriate curves here given.

St. John's College,
Feb. 1850.

ERRATA.

Page	Line	
10	12 and 13	*for* α *read* a.
19	24	*for* sector *read* section.
35	19	*for* tan ( ) *read* $\log_e$ tan ( ).

PART I.

ON THE OBLIQUE BRIDGE CONSTRUCTED WITH SPIRAL COURSES.

CHAPTER I.

1. THE inferior and superior surfaces of an arch are called respectively the *intrados* (or soffit), and the *extrados*. The continuous surfaces, generally covered with mortar or cement, which divide successive courses of arch stones, are the *beds* of the arch stones. The curved lines in which these beds meet the extrados and intrados are called respectively the *extradosal* and *intradosal coursing joints*. The other short discontinuous joints are the *heading joints*. The vertical planes which limit the length of the arch are called the *faces* of the arch, and those arch stones, which are cut by these planes, are the *quoins* of the arch. The abutments are the upright walls which bound the width of the road under, and support the arch for carrying the road over, the bridge. A section of the arch made by a plane at right angles to the axis of the intrados is a *right section*, commonly called a "section on the square."

2. A cylindrical surface may be supposed to be generated by a straight line moved parallel to a given fixed straight line, so as always to pass through a given curve called the *directrix*. It is evident that any curve that could be traced on the surface of a cylinder, provided it made a

complete circuit, might be considered to be the directrix of that cylinder, but in all cases where we have occasion to employ the term *directrix,* we shall consider it to be a curve in one plane at right angles to the straight line to which the generating line is always parallel, and consequently it will denote a section of the cylindrical surface " on the square."

3. Cylindrical surfaces are developable; that is, if we suppose a thin, flexible, and inextensible sheet to coincide with the surface of a cylinder, this sheet admits of being extended on a plane without being rumpled or torn. After the development of the soffit of an arch has been obtained the arrangement of the courses is easily determined.

4. Before commencing the working drawings for an oblique bridge, the acute angle (Ω) between the directions of the roads over and under the proposed bridge must be found, as well as the widths of those roads. The width of the road is the distance of the walls on each side, measured in a direction at right angles to that of the road. If *bc,* Fig. 4, be the plan of an oblique arch, *ae* will be the width of the road under, and *am* the width of the road over, together with the thickness of the two parapets. In determining the form of the arch, the attention should be directed to the form of the directrix, or "section on the square," for upon this depends the size of the vehicles that can pass beneath the arch.

5. Fig. 1. is an isometrical projection of a direct arch, the faces *ale,* and *bm'* being at right angles to the axis of the cylinder, and consequently to the generating lines of the cylinder which are parallel to the axis. The development of the surface *ble* will be the rectangle *be',* the sides of which are respectively equal to the length of the abutment *ab,* and the arc *ale* of the directrix. The axis of the soffit will always be supposed to be horizontal.

6. Let the face *aei'* in Fig. 1 revolve about the vertical line *aj* through an angle = 90° – Ω, and come into the position *jac,* Fig. 2. The directrix of the cylinder ap_1p_2e remains the same as before, and when the surface of the cylinder is

developed becomes the straight line aq_1q_2e', at right angles to ab and all the generating lines. p_1p_1', p_2p_2', &c., are portions of the generating lines of the cylinder cut off by the planes of the face $ap_1'c$, and of the directrix ap_1e.

7. From a in ab, Fig. 2, draw ae' at right angles to ab, and take aq_1, aq_2, ... ae' respectively equal to the arcs ap_1, ap_2,...ae of the directrix. Through q_1, q_2...e' draw straight lines parallel to ba, and make $q_1\gamma_1$, $q_2\gamma_2$...$e'c$ respectively equal to p_1p_1', p_2p_2'...ec, then $a\gamma_1\gamma_2$...c will be points in the development of $ap_1'p_2'$...c. If the faces aci, bm of the bridge be parallel planes, the curve bmd will be the same as alc, and dc will be equal and parallel to ab.

8. Suppose the development to be formed on the horizontal plane cab passing through the tops of the abutments. From p_1p_2..., $p_1'p_2'$... let fall the perpendiculars $p_1\alpha_1$, $p_2\alpha_2$,... $p_1'\beta_1$, $p_2'\beta_2$, ... meeting the plane bac in the points $\alpha_1\alpha_2$..., $\beta_1\beta_2$...respectively. Join $\alpha_1\beta_1$, $\alpha_2\beta_2$...which will be equal and parallel to p_1p_1', p_2p_2', ... respectively. As before, in ae' drawn at right angles to ab, make aq_1 = arc ap_1, aq_2 = arc ap_2, &c. Through q_1q_2... draw $q_1\gamma_1$, $q_2\gamma_2$,... parallel to ab, and through $\beta_1\beta_2$... draw $\beta_1\gamma_1$, $\beta_2\gamma_2$, &c., at right angles to ba, and produce them so as to intersect $q_1\gamma_1$, $q_2\gamma_2$ ... in $\gamma_1\gamma_2$... respectively. Then it is manifest as before that $\gamma_1\gamma_2$...are required points in the development of the edge of the cylinder alc; for $q_1\gamma_1 = \alpha_1\beta_1 = p_1p_1'$; $q_2\gamma_2 = \alpha_2\beta_2 = p_2p_2'$, &c.

9. The following modification of the above process is very commonly used in obtaining developments of the soffits of oblique arches. The directrix $al'e$ and chord ae are traced out on paper, Fig. 3. Draw ac making an angle $90° - \Omega$ with ae; and av and ec at right angles to ae. Produce ea and make ae' = arc $al'e$. Divide ae' and $al'e$ into any the same number of equal parts in q_1, q_2, q_3...; p_1, p_2, p_3,... respectively; and through these points draw lines $q_1\gamma_1$, $q_2\gamma_2$...; $p_1\alpha_1\beta_1$, $p_2\alpha_2\beta_2$...at right angles to $e'ae$. Again through β_1, β_2, β_3,...draw lines parallel to eae' cutting the lines $q_1\gamma_1$, $q_2\gamma_2$,...in γ_1, γ_2...which are points in the required curve. Take ab, $cc'd$ each at right angles to eae' and equal to the

length of the abutments. Draw the curve bmd equal in all respects to alc, then $abmdcl$ is the required development of the soffit of the arch.

10. If the development of the extrados be required, draw the chord EA' of $A'L'E$ the right section of this surface, produce it to E', and make $A'E' =$ arc $A'L'E$, Fig. 3. Through E draw EC at right angles to $A'E$, and through A' draw $A'C$ inclined at an angle $90° - \Omega$ to $A'E$, or parallel to ac, so as to meet EC in C. $A'L'E$ and $A'E'$ must be divided into the same number of equal parts; straight lines must be drawn through all these points of division parallel to EC, and from the points where $A'C$ is cut other straight lines must be drawn parallel to $A'E$, and the extradosal heading-spiral will then be determined precisely as $c'la$ was found.

11. If upon av a curve $a\delta_4 v$ be described *similar* to the figure $al'e$ described upon ae, then it is manifest that if $ap_1 = p_1p_2 = p_2p_3 =$ &c., that $a\delta_1 = \delta_1\delta_2 = \delta_2\delta_3 =$ &c. Hence the following method of finding the development. In Fig. 6 make $ca = ce$ in the plan Fig. 4, draw Oe' bisecting ac at right angles in O, and make $Oe' =$ arc $al'e$, Fig. 3. Upon ca describe the curve $cl'a$ similar to the right section $al'e$. Divide the straight line Oe' and the arc $cl'a$ into the same number of equal parts, draw lines as indicated in Fig. 6, and thus determine the points 1, 2, 3, &c. in the required curve ala'.

12. The method of obtaining the development of a cylindrical surface, in Arts. 8 and 9, is independent of any consideration of the form of the directrix $al'e$, or of the line ac in the plan. Henceforth ac will be supposed to be a straight line, and $al'c$ an arc of a circle not greater than a semicircle, as it appears that such bridges are quite sufficient for all practical purposes.

13. When an oblique bridge is constructed with spiral courses, the right section should *in no case* approach near to a semicircle, although such are sometimes seen: Figures 8—17 relate to such a case. Fig. 18 represents the mode of

obtaining the development of the intrados when the right section of the arch is an arc of a circle less than a semicircle. Let O be the centre of $A'L'E$, and $al'e$ then OaA', OeE are straight lines. Having joined ea and produced it to e', make ae' = arc $al'e$, and divide them into the same number of equal parts in $q_1q_2q_3$ &c. $p_1p_2p_3$ &c., and proceed as before to determine the points $\gamma_1\gamma_2\gamma_3$ &c. in the intradosal heading spiral. In the same way may the development of the extradosal heading spiral be found by first joining EA', producing it to E', and making $A'E'$ = arc $A'L'E$.

14. Suppose that the right section $al'e$ subtends at the centre O an angle 2α. Take an in the plan Fig. 4 equal to Oa, the radius of the directrix Fig. 18, and draw nn' parallel to dc or ba. As in Art. 11, make ac, Fig. 7, equal to ec, Fig. 4. Draw $e'O$ bisecting ac at right angles in O with centre O' in $e'O$ or $e'O$ produced, and radius nn', Fig. 4, describe the arc of a circle passing through a and c. Make Oe' equal to the arc $al'e$, Figs. 18 and 18*, divide Oe' and arc ca into the same number of equal parts in 1″, 2″, 3″, &c. 1′, 2′, 3′, &c. and determine the points 1, 2, 3, &c. in the manner indicated in Fig. 7, where the $\angle\, aO'c = 2\alpha = \angle\, aOc$, Figs. 18 and 18*. For a still more convenient method of determining developments, see Figs. 61 and 65. Fig. 19 exhibits so much of the development as will be required when full-sized drawings are employed.

15. After having obtained the development of the intrados, Fig. 3, we must proceed to determine the coursing joints. The method generally adopted is as follows, when spiral courses are employed. Join bd, ac, Fig. 8, and then bc is a parallelogram. Divide the equal sides ac, bd into the same number (generally odd) of equal parts, according to the thickness of the intended courses. From a let fall the perpendicular an upon bd. Suppose that n does not coincide with one of the points of division, and that it falls nearest to r; then join ar, and through all the other points of division in db, ca, draw lines parallel to ar, terminated both ways by the heading spirals dmb, cla, or by the abutments ab, cd, and

thus the intradosal coursing joints will be determined. The short transverse lines at right angles to *ar*, as $a\gamma$, $\beta\delta$, &c., are called heading joints, and divide the courses into arch stones.

16. The curved lines *bmd*, *alc*, cut the parallel generating lines at all angles varying from 90° at the abutments *a*, *b*, *c*, *d*, Fig. 8, to (Ω) at the middle points *l* and *m*, and it is consequently impossible to draw a series of parallel lines meeting these curves at right angles. The extremities *b*, *d* of the curve *bmd* are joined, and the coursing joints are drawn at right angles, or approximately at right angles to *db*. By slightly varying the length of the abutments *ab* and *cd*, the foot of the perpendicular *n* might be made to coincide exactly with one of the divisions, as *r* of *db*, but such nicety is not necessary for the stability of the structure.

17. The straight line *lm* divides all the developments of the intrados into two equal portions: it is the line in which a vertical plane passing through the axis of the cylinder would intersect the development *acdb* when returned to its proper cylindrical form. *LM* is the corresponding line for the extrados: *lm*, *LM* are also equal in length, for they are parallel lines cut off by two parallel planes, namely the faces of the arch. See Figs. 50, 52, and 54.

18. Produce *ar*, *cd*, Fig. 8, to meet in *h*, and draw *hk* at right angles to *ch* meeting *ab* produced in *k*. Fig. 9 represents the development of the extrados placed so that, if *mM*, *lL* were joined, *mM Ll* would be a rectangle. Produce *hk* to meet *CD*, *AB*, produced, in *H*, *K* respectively. From *a* let fall the perpendicular *aA'* on *BA'* and join *A'H*. Then *A'H* is the direction of the coursing joint on the extrados corresponding to *ah* on the intrados, Fig. 8.

19. Let $B_0p_0P_0$ be a straight line at right angles to $A_0B_0C_0$, Fig. 9*, and let p_0, P_0, be two points in B_0P_0, such that $B_0p_0 = r =$ the radius of the intrados *al'e*, Fig. 3, and $B_0P_0 = R =$ the radius of the extrados *A'L'E*. Suppose that A_0C_0 coincides with the common axis of the cylinders, and that it is caused to move *uniformly* in direction of its length,

whilst the arm B_0P_0 revolves *uniformly*, the velocities being so adjusted that B_0 would describe ak in the same time that B_0p_0 would revolve through an angle equal two right angles, Fig. 3, then the point p_0 will trace out the same helix as would be formed by ah, if ak were applied along the generating line of a cylinder of radius r, and kd were wrapped round the cylinder.

20. In like manner P_0 would trace out the helix that would be formed by HK if KHA' were formed into a cylindrical surface of radius R, KA' being parallel to the axis of the cylinder. For $A'K = ak$, and $KH = A'L'E$ = semicircle to radius R.

21. The angle $cha = hak$ is the angle in which the intradosal coursing joints intersect the generating lines of the cylinder, and we shall always denote this angle by ϕ. In like manner the extradosal angles DHA' we shall denote by Φ.

22. Suppose that $ak = A'K = L$, $kh = \frac{1}{2}$ circumference of a circle of radius $r = \pi r = 3.14159 \times r$, and $KH = \frac{1}{2}$ circumference of a circle to radius $R = \pi R = 3.141.59 \times R$. Now

$$\tan\phi = \frac{kh}{ak} = \frac{\pi r}{L}; \text{ and } \tan\Phi = \frac{\pi R}{L},$$

$$\therefore \frac{\tan\Phi}{\tan\phi} = \frac{\pi R}{L} \div \frac{\pi r}{L} = \frac{\pi R}{L} \times \frac{L}{\pi r} = \frac{R}{r}.$$

23. Produce LM to meet $A'H$ in T, and lm to meet ah in t, Figs. 8 and 9. It is evident that if the equidistant coursing joints be produced in Fig. 8, they will intersect tl in $a_1a_2a_3$...such that $ta_1 = a_1a_2 = a_2a_3$ = &c. It is also evident that if the corresponding extradosal joints be produced, they will intersect TL in equidistant points $\beta_1\beta_2\beta_3$ &c.; so that $ta_1 = a_1a_2 = a_2a_3$ = &c. = $T\beta_1 = \beta_1\beta_2 = \beta_2\beta_3$ = &c. If we thus determine the points $\beta_1\beta_2\beta_3$...and draw lines parallel to TA' through $\beta_1\beta_2\beta_3$ ... we shall determine the coursing joints on the extrados corresponding to those already found on the intrados.

24. In Fig. 25 the line *lm* is placed upon *LM*, the point *l* being made to coincide with *L*, and consequently $a_1a_2a_3$ &c. with $\beta_1\beta_2\beta_3$ &c. If through any point a_4 or β_4 straight lines be drawn inclined respectively at angles ϕ, Φ, to *lm*, these will represent corresponding intradosal and extradosal coursing joints on the developments.

25. We now proceed to consider how the projections of these coursing joints on a horizontal plane and on various vertical planes may be obtained. The projecting lines will always be supposed to be perpendicular to the plane of projection. First, to find the projection of the intradosal coursing joint *ah*, Fig. 8, on a horizontal plane, when that development has been returned to its proper cylindrical form to radius *r*. In Fig. 10 describe the right section *al′e* of the intrados, as in Fig. 3, and draw the chord *ae*, which must be bisected in *w*. Draw ao_{10}, *wv*, and *ek* at right angles to *ae*, making $wv = L$. Divide the arc *al′e* into any number of equal parts in $p_1p_2p_3$ &c. and also divide *wv* into the same number of equal parts in $a_1a_2a_3$ &c. Draw lines through the above-mentioned points to intersect (two and two), as indicated in Fig. 10, and thus the points $o_1o_2o_3$ &c. in the required projection will be found: for it is manifest that if we suppose B_0, in Fig. 9*, to coincide with *w*, and the arm B_0p_0 with *we*, then B_0 will move in the direction *wv* from *w* to a_1 in the same time as p_0 describes an arc equal to ep_1 by reason of the rotation of the arm B_0p_0. Hence it is evident that o_1 is a point in the curve sought. Again, by the time that B_0 arrives at a_2, p_0 will have revolved through an arc equal to ep_2, and therefore o_2 is another of the points sought, and so on.

The projection of a given curve is the same on all horizontal planes, and hence we may suppose the horizontal plane of projection to pass through the tops of the abutments *ab*, *cd*.

26. To find the projection of the coursing joints on a vertical plane parallel to the axis. In Fig. 13 take $wv = ak = L$, Fig. 8, and describe on *vw* the right section of the intrados *al′e*. As before, divide *vw* and arc *al′e* into the same

number of equal parts in the points $a_1 a_2 a_3$ &c. $p_1 p_2 p_3$ &c. Through a_7 and p_7 draw lines respectively perpendicular and parallel to *wv* intersecting in γ_7. Similarly for each pair of corresponding points a_1, p_1; a_2, p_2, &c., and thus the points $\gamma_1 \gamma_2 \gamma_3$ &c. in the required projection will be obtained. For it is evident that B_0 would have moved from *w* to a_1 by the time that p_0 had described the arc ap_1, and consequently that the point γ_1 would be the required projection of p_0 at this instant. So also when B_0 had arrived at a_2, p_0 would have described the arc ap_2, and γ_2 would be its projection. Hence $\gamma_1 \gamma_2 \gamma_3$ &c. are points in the required projection of the helix.

27. Let it be required to determine the projection of the intradosal coursing joints on any vertical plane that intersects the horizontal plane of projection in *VW*, Fig. 11. Describe the right section *al'e* of the soffit, so that the points *a*, *e* may be in the straight line *VW* or *VW* produced. Divide *al'e* into exactly the same number of equal parts in $p_1 p_2 p_3$ &c. as in Fig. 10. From $o_1 o_2 o_3$ &c. let fall perpendiculars on *VW*, and produce them to intersect in $\delta_1 \delta_2 \delta_3$ &c. the lines drawn through $p_1 p_2 p_3$ &c. parallel to *WV*, then $\delta_1 \delta_2 \delta_3$ &c. will be points in the required projection. The plane $V\delta_1 \delta_2 \delta_7 W$ is supposed to be perpendicular to the plane of the paper. It has been already shewn that when B_0 comes to a_1, p_0 will be in a vertical line passing through o_1 at a distance $p_1 r_1$ above it. When p_0 is in a vertical line passing through o_2, it is situated a space $p_2 r_2$ above o_2 &c. Fig. 12 exhibits the projection of the same curve on a vertical plane at right angles to the plane of projection in Fig. 11.

28. The mode of obtaining the projections of the coursing joints of the extrados is the same as that which has been applied to find those of the intrados: $vw = L$ remains the same, and the right section of the extrados *A'L'E* must be used instead of *al'e*, Fig. 10. The projections on the horizontal plane of *ah* and *A'H* are shewn in Fig. 15. The projections of the intradosal and extradosal joints for *any given course* always cut *vw* in the same point as at *t*.

29. In Fig. 16 *ath* is the projection of the intradosal coursing joint *ath*, Fig. 8, on the horizontal plane. If in Fig. 16 we determine the points $a_1a_2a_3$ &c. so that they are precisely at the same distances from t as in Fig. 8, and if through these points $a_1a_2a_3$ &c. curves identical in form with *ath*, and similarly situated with respect to OO', be drawn, as a_1a_1', a_2a_2' &c., these will be the projections of the intradosal coursing joints on the horizontal plane. Make the $pac = 180^\circ - \Omega$, then $tO = tl$ in Fig. 8. Make *ab* equal to the length of the abutment, and draw *bd* parallel to *ac*; then *acdb* will include the projections of the courses actually used in the erection of the bridge, and $a_1'a_2'a_3'$ &c. will be the projections on a horizontal plane of the points $a_1a_2a_3$ &c. Fig. 49, in which the intradosal helices meet the face of the arch.

30. Through h draw *hp* parallel to *ac*, and produce *wv* to meet *ph* in O'; then if OO' be divided into the same number of equal parts as *bd* or *ac* was divided into in Fig. 8, each of these will be $= ta_1 = a_1a_2 = a_2a_3 =$ &c.

31. In Fig. 17 *ath* is the projection of the intradosal coursing joint on the horizontal plane. Divide OO' into 27 equal parts (the same as the number of quoins in the arch, which in this particular case is 27), and let $a_1a_2a_3$ &c. be the points of division. Through $a_1a_2a_3$ &c. draw lines $a_1\lambda_1$, $a_2\lambda_2$, $a_3\lambda_3$, &c. parallel to *ac* meeting *ath* in $\lambda_1\lambda_2\lambda_3$ &c., and again through $\lambda_1\lambda_2\lambda_3$ &c. draw the lines λ_1a_1', λ_2a_2', λ_3a_3', &c. parallel to $O'O$ meeting *ac* in $a_1'a_2'a_3'$ &c., and it is manifest that $a_1'a_2'a_3'$ &c. are at the same distances from O in Fig. 17 as they are when determined as in Fig. 16. Upon *ac* as axis major, with r for semi-axis minor, describe the ellipse *alc*, and from $a_1'a_2'a_3'$ &c. draw lines at right angles to *ab* so as to intersect *cla* in $a_1a_2a_3$ &c. Then $a_1a_2a_3$ &c. are the points on the elevation of the bridge, Fig. 49, determined so as to agree with the varying distances of the points $a_1a_2a_3$ &c. in the development Fig. 8.

32. If OO' in Fig. 15 be divided into as many equal parts in $a_1a_2a_3$ &c. as there are quoins in the elevation Fig. 49, and if through these points lines be drawn parallel to *ac* cutting

the projections of the extradosal and intradosal coursing joints in $\gamma_1\gamma_2\gamma_3$ &c. $\lambda_1\lambda_2\lambda_3$ &c. respectively, then $\lambda_1\gamma_1$, $\lambda_2\gamma_2$, $\lambda_3\gamma_3$, &c. are the magnitudes of the projections of the joints a_1b_1, a_2b_2, a_3b_3, &c. in Fig. 49. Upon ac and AC as major axes, with minor axes $2r$ and $2R$, describe the semi-ellipses cla and CLA. Through γ_5 and λ_5 draw straight lines parallel to vw cutting ac in a_5' and b_5'; and from these points draw $a_5'a_5$ and $b_5'b_5$ at right angles to AC cutting the ellipses in a_5 and b_5. Join b_5a_5, and this will be the approximate position of the joint as it appears on the face of the arch, Fig. 49.

33. The points $a_1a_2a_3$ &c. $b_1b_2b_3$ &c. in the elevation of the arch corresponding to the points in which the equidistant parallel coursing joints in the developments Figs. 8 and 9 meet the heading-spiral, will be more correctly determined by the methods indicated in Figs. 15 and 17 than by that in Fig. 16. In most of the ordinary cases that occur in practice, it will be perhaps the best method to take a chord with the compasses so small that it may be supposed to be equal to the corresponding arc, and then step along the heading-spiral alc, Fig. 8, and thus divide it into approximately equal parts. Commence from the corresponding point on the ellipse alc, Fig. 15, with the compasses unmoved, and divide it also in the same manner. The length of the heading-spiral alc, Fig. 8, is the same as the elliptic arc alc, Figs. 15, 16, 17; and the distance of a joint as a_5 can be found from the division that falls nearest it on the development, and the same distance can be immediately measured from the corresponding point in the elevation. Figs. 18 to 25 have been given as illustrations of the modes of finding the developments of cylindrical surfaces, and the projections of the coursing joints when the right section of the arch $al'e$ is a segment of a circle less than a semicircle. Let $\angle\, aOe$, Fig. 18, be denoted by $2a$, and let $wv = ak = L$. Here L denotes the space through which B_0, Fig. 9*, must move uniformly in the same time as B_0p_0 revolves uniformly through an angle $= 2\alpha$. If $2\alpha = \pi$, this case becomes identical with the preceding. We have supposed the plan and the obliquity Ω to remain unaffected. We still

denote the radii of the intrados and extrados by r and R. Also ϕ and Φ denote the angles $e'ha$ and $E'HA'$, Figs. 20 and 21, which are the angles of the intrados and extrados.

34. $$\text{Tan}\ \phi = \frac{ae'}{e'h} = \frac{2ar}{L};\ \tan\Phi = \tan A'HE' = \frac{A'E'}{E'H} = \frac{2Ra}{L};$$

and therefore, as before, $\dfrac{\tan\Phi}{\tan\phi} = \dfrac{2Ra}{L} \div \dfrac{2ra}{L} = \dfrac{R}{r}$.

35. We have supposed the straight line ac, Fig. 20, to be divided into 19 equal parts, or that there are 19 quoins on each face of the arch: hence OO', Fig. 24, must be divided into 19 equal parts, when it is desired to determine the points $a_1'a_2'a_3'$ &c., and thence $a_1a_2a_3$ &c.

36. Mr. Buck was the first to observe, that in carefully-executed drawings if b_5a_5, Fig. 15, be joined and produced to intersect the axis minor lO produced in Q, then all other joints b_1a_1, b_2a_2, b_3a_3 &c. being produced will pass through the same point Q. Mr. Buck also gave an algebraical expression for the determination of this point, but the following construction will be found far more easy of application. If the right section of the arch be a segment of a circle, as in Fig. 18, complete the semi-ellipses $A'LC'$, $a'lc'$, Fig. 26, of which ALC, alc are arcs. With the semi-axis major Oc' of the *intradosal* ellipse as radius and l as centre, describe an arc cutting Oc' in s. At s in Os make the angle OsQ equal to Φ, the angle of the extrados, and the point Q, where this line meets the axis minor of the ellipse lO produced, is the focus to which all the joints on the face approximately converge. Or, with L as centre and radius equal OC', describe an arc cutting OA' in S, and at the point S in OS make the angle OSQ equal ϕ, the angle of the intrados, then SQ will intersect LO produced in the same point as sQ did. See Art. 45.

37. The above is a very useful property, and enables us in practice to dispense with many of the drawings that it has been necessary to give in order to furnish a complete account of the mode of constructing the oblique bridge with spiral courses. Suppose that in Figures 14 and 26 Q has been

determined, and also a'' the point in which one of the intradosal coursing joints meets the face of the arch. Join Qa'' and produce it to cut the extradosal ellipse in b''; then $b''a''$ is the approximate position of the intersection of the bed joint and the face of the arch. Consequently it will be sufficient to determine the points $a_1a_2a_3$ &c. in the manner shewn at Figs. 17 and 25, and then, knowing Q, $b_1b_2b_3$ &c. will be easily found.

38. The joints that are seen on the face of an arch a_1b_1, a_2b_2, &c. formed by the intersection of the twisted beds of the stones and the face, are lines slightly curved towards the keystone lL, but for all practical purposes it is sufficient to consider them to be straight lines. That mode of determining Q which is most convenient in any particular case must be adopted. The value of Φ may be found from the equation $\tan \Phi = \frac{R}{r} \tan \phi$; or, Fig. 8*, draw $a'b'$ parallel to hk, and make $a'b' = r$. Also through b' draw $b'c$ parallel to dh meeting $a'h$ in c. Make $b'd' = R$, and join cd'. Then the angle $b'cd'$ $= \Phi$, for we have the angle $b'ca' = \phi$, and

$$\tan b'cd' = \frac{d'b'}{cb'} = \frac{d'b'}{a'b'} \frac{a'b'}{b'c} = \frac{R}{r} \tan \phi = \tan \Phi, \therefore b'cd' = \Phi.$$

CHAPTER II.

39. From the practical methods of obtaining the developments and projections given in the preceding Chapter, it is easy to deduce the equations to those curves. Thus in Fig. 3, suppose that ae' and ab are the axes of y' and x' and $aq = y'$, $q\gamma = -x'$, $\angle aOp = \theta'$: then

$$y' = aq = \text{arc } ap = r\theta',$$

$$-x' = q\gamma = a\beta = a\alpha \cot\Omega = r \text{ vers } \theta' \cot\Omega,$$

$$\left.\begin{aligned} y' &= r\theta' \\ x' &= -r\cot\Omega \text{ vers } \theta' \end{aligned}\right\}\ldots\ldots\ldots\ldots\ldots(1).$$

40. It will generally be found more convenient to make the middle point l the origin of coordinates; the new axes xx', yy' being taken parallel to the old. Let $lr = y$, $r\gamma = -x$, and $\angle l'Op = \theta$: then

$$y = lr = qq_6 = qa - q_6a = \text{arc } pa - \text{arc } p_6a = \text{arc } pp_6 = r\theta$$

$$-x = r\gamma = \beta s = \beta_6 s \cot\Omega = O\alpha \cot\Omega = r\cos\left(\frac{\pi}{2} - \theta\right)\cot\Omega$$

$$= r\cot\Omega\sin\theta;$$

$$\therefore \left.\begin{aligned} y &= r\theta \\ x &= -r\cot\Omega\sin\theta \end{aligned}\right\}\ldots\ldots\ldots\ldots\ldots\ldots\ldots(2),$$

are the required equations to the heading-spiral alc, where the origin is at the middle point l, and the axes of x and y are respectively parallel and at right angles to the generating lines of the cylindrical surface.

In like manner the equations to the development of the extradosal heading-spiral ALC, Fig. 9, will be found to be

$$\left.\begin{aligned} Y &= R\theta \\ X &= -R\cot\Omega\sin\theta \end{aligned}\right\}\ldots\ldots\ldots\ldots\ldots\ldots(3).$$

41. Let L denote the space uniformly described by the point B_0 in the line A_0C_0, Fig. 9*, whilst the arm B_0P_0 is

revolving uniformly through an angle $2a$, as in Art. 33, where $2a$ is the angle aOe, Fig. 18*. Suppose that when B_0 is at o_5, Fig. 10, the arm B_0P_0 coincides with the axis of z which is vertical. The axis of y coincides with the axis of the cylinder, and the axis of x is xx' in the same horizontal plane with the axis of y, and at right angles to it. Suppose that when B_0 has described a space $y = o_5a$ that the arm B_0P_0 has revolved through an angle θ. Then we get

$$y : L :: \theta : 2a; \text{ or } y = \frac{L\theta}{2a} = \mu\theta, \text{ if we put } \mu \text{ for } \frac{L}{2a},$$

$$-x = ao = wr = r\cos\left(\frac{\pi}{2} - \theta\right) = r\sin\theta,$$

$$z = pr = r\sin\left(\frac{\pi}{2} - \theta\right) = r\cos\theta.$$

Hence if we substitute for θ any numerical value in the equations

$$y = \mu\theta,\ x = -r\sin\theta,\ z = r\cos\theta \ldots\ldots\ldots\ldots\ldots (4),$$

we shall obtain the coordinates of a point in the intradosal coursing joint. By substituting in (4) different values of θ the coordinates of any number of points may be obtained in that curve.

42. In like manner, the same axes being retained, it may be shewn that the equations to the corresponding extradosal coursing joint are

$$Y = \mu\theta,\ X = -R\sin\theta,\ Z = R\cos\theta \ldots\ldots\ldots (5).$$

The equations to the projections of these helices on a horizontal plane are

$$x = -r\sin\theta,\ y = \mu\theta, \text{ Figs. 10 and 22} \ldots\ldots\ldots (6),$$

$$X = -R\sin\theta,\ Y = \mu\theta \ldots\ldots\ldots\ldots\ldots\ldots\ldots (7).$$

The equations to their projections on a vertical plane parallel to the abutments are

$$y = \mu\theta,\ z = r\cos\theta, \text{ Figs. 13 and 23*} \ldots\ldots\ldots (8),$$

$$Y = \mu\theta,\ Z = R\cos\theta \ldots\ldots\ldots\ldots\ldots\ldots\ldots (9).$$

43. The equation to a vertical plane parallel to the face of the bridge, cutting the axis of y at a distance d from the origin, is

$$y - d = x \cot \Omega \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (\alpha):$$

required the coordinates $x'y'z'$, $X'Y'Z'$ in which this plane is pierced by the intradosal and extradosal helices

$$\left.\begin{aligned} y &= \mu\theta \\ x &= -r\sin\theta \\ z &= r\cos\theta \end{aligned}\right\} \ldots(\beta) \quad \left.\begin{aligned} Y &= \mu\Theta \\ X &= -R\sin\Theta \\ Z &= R\cos\Theta \end{aligned}\right\} \ldots(\gamma).$$

Now the point $x'y'z'$ is a point in the helix (β) and in the plane (α). Hence we have four equations for finding $x'y'z'$ and θ. In like manner we have four equations (γ) and (α) for finding $X'Y'Z'$ and Θ. Substituting the values of x and y in (β) for x and y in (α) we get $\mu\theta - d = -r\sin\theta \cot\Omega$ (δ). So also $\mu\Theta - d = -R\sin\Theta \cot\Omega$ (ϵ), which are the two equations for the determination of θ and Θ, but unfortunately they can be solved only by continual approximation. When θ and Θ have been obtained, it remains only to substitute their values in equations (β) and (γ) to find $x'y'z'$, $X'Y'Z'$.

44. Suppose that the two points $x'y'z'$, $X'Y'Z'$ of last article are on the face of the bridge, the equation to the projection on (xz) of the straight line joining them is

$$z - z' = \frac{Z' - z'}{X' - x'}(x - x').$$

In Figs. 14 and 26 let $x'y'z'$ be the coordinates of a'', and $X'Y'Z'$ those of b''. If then in the above equation we make $x = 0$, $z = OQ$,

$$\therefore OQ - z' = \frac{Z' - z'}{X' - x'}(0 - x');$$

$$\text{or } OQ = \frac{z'X' - x'Z'}{X' - x'},$$

$$= \frac{r\cos\theta \times (-R\sin\Theta) - (-r\sin\theta)R\cos\Theta}{-R\sin\Theta + r\sin\theta},$$

$$= \frac{Rr\left\{\sin\Theta\cos\theta - \cos\Theta\sin\theta\right\}}{R\sin\Theta - r\sin\theta}.$$

But by (δ) and (ϵ)

$$\mu\Theta - d = -R\sin\Theta\cot\Omega, \text{ and } \mu\theta - d = -r\sin\theta\cot\Omega;$$

$$\therefore \mu(\Theta - \theta)\tan\Omega = -(R\sin\theta - r\sin\theta),$$

$$\therefore OQ = \frac{-Rr}{\mu\tan\Omega}\,\frac{\sin(\Theta - \theta)}{(\Theta - \theta)},$$

$$= -\frac{Rr}{\mu\tan\Omega} \text{ nearly, } \ldots\ldots\ldots\ldots\ldots (10),$$

since the difference of Θ and θ is always so small that we may consider $(\Theta - \theta)$ to be equal $\sin(\Theta - \theta)$. Hence it appears that OQ is constant for any particular oblique arch; and if we find Q and a'' on the intradosal ellipse lc, join Qa'' and produce it to intersect the extradosal ellipse LC in b'', then $a''b''$ will be the approximate position of the joint on the face of the arch. See Figs. 49—52.

45. In Art. 41 μ was written for the constant $\frac{L}{2a}$, and hence

$$OQ = -\frac{Rr}{\mu\tan\Omega} = -Rr\frac{2a}{L}\cot\Omega = -R\left(\frac{2ra}{L}\right)\cot\Omega$$

$$= -R\frac{ae'}{e'h}\cot\Omega, \text{ Figs. 8 and 20, } = -R\tan\phi\cot\Omega;$$

$$\text{or, } OQ = -r\left(\frac{2Ra}{L}\right)\cot\Omega = -r\frac{HK}{A'K}\cot\Omega, \text{ Figs. 9 and 21,}$$

$$= -r\tan\Phi\cot\Omega.$$

46. Now the axis major of the intradosal ellipse is always $2r\operatorname{cosec}\Omega$, and the axis minor $2r$. The axes of the extradosal ellipse are $2R\operatorname{cosec}\Omega$ and $2R$. With centre L and radius $LS = OC = R\operatorname{cosec}\Omega$, describe an arc cutting OC, the axis major in S, which is the focus of the ellipse LC. Hence, in Fig. 14, $LS = OC = R\operatorname{cosec}\Omega$; $LO = R$, and $\angle SOL$ is a right angle;

$$\therefore \cos SLO = \frac{LO}{LS} = \frac{R}{R\operatorname{cosec}\Omega} = \sin\Omega = \cos(90° - \Omega);$$

and $\therefore \angle SLO = 90° - \Omega$; or $\angle OSL = \Omega$. From S draw SQ, making with OS the angle $OSQ = \phi$, and let it intersect LO produced in Q, then Q is the point to which the joints on the face of the arch converge, as in Figs. 49 and 51. For $OQ = OS \tan \phi = (OL \cot \Omega) \tan \phi = R \cot \Omega \tan \phi$. The negative sign in Art. 45 implies that OQ must be measured downwards from O.

Again, $ls = Oc = r \operatorname{cosec} \Omega$, and $lO = r$;

$$\therefore \cos Ols = \frac{Ol}{ls} = \frac{r}{r \operatorname{cosec} \Omega} = \sin \Omega;$$

$$\therefore \angle\, Ols = 90^\circ - \Omega, \text{ and } \angle\, Osl = \Omega.$$

From s draw sQ, making with sO the angle $OsQ = \Phi$, and meeting lO produced in Q: this will be the same point as was found above, for

$$OQ = Os \tan \Phi = (r \cot \Omega) \tan \Phi. \quad \text{See Art. 45.}$$

If the right section of the bridge be less than a semicircle, as in Fig. 18, then the arcs AC, ac, Fig. 26, must be produced both ways so as to complete the semi-ellipses $A'LC''$, $a'lc'$ and then the focus of the joints Q may be found as above. This explains the construction given in Art. 36.

47. To find the angle which the tangent rt to the intradosal ellipse apl, Fig. 58, at the point p makes with the generating line pp' passing through that point. OO' is the axis of the cylinder, rOr' is the horizontal plane in which it is situated, and Ot is vertical. $a'lc'$ is a right section, and alO is a section made by a vertical plane passing through tO, and inclined at an angle $aOa' = 90^\circ - \Omega$ to the right section $a'lc'$. Draw a tangent plane to the surface of the cylinder at the point p, then this plane will touch the surface of the cylinder along the generating line pp', and it will also cut the horizontal plane aOa' in rr', a line parallel to pp' and therefore to OO'. The tangent plane will be cut by the vertical cutting planes tOr, tOr' in the straight lines rpt, $r'p't$, which will be tangents at the points p, p' to the ellipse apl and circle $a'p'l$. Let fall the perpendiculars pn, $p'n'$ on the plane rOr', and join $n'n$, which will be equal and parallel to pp', and also parallel to OO'. Also the angle $pp't$ is a right angle, because pp' is parallel to the axis OO', and is therefore at right angles to the right section $a'lc'$ and to all straight lines in that plane, and the

angle $tpp' = \psi$ is the angle required. In Fig. 59, $a'lc'$ is the right section, and apt the intradosal ellipse; p, p' are the two points respectively in the ellipse and circle at the same height above the horizontal plane aOc'. Fig. 60 is a plan of Fig. 58,

$$\tan\psi = \frac{p't}{pp'}, = \frac{1}{pp'}\left(p'm \,.\, \frac{Op'}{Om}\right) \text{ by similar triangles } p'mt,\ Omp',$$

$$= \frac{1}{nn'}\left(n'O \times \frac{r}{Om}\right) = \frac{n'O}{n'n} \times \frac{r}{Om},$$

$$= \frac{r}{Om}\tan\Omega.$$

48. ALC, alc, Fig. 63, are the extradosal and intradosal ellipses for the example where the right section is a semicircle, Fig. 3. At the extremity l of the axis minor make the angle $OlR = \Omega$, and from the point R in which lR intersects the axis major draw RG at right angles to ac. From any point p in the ellipse alc draw pm parallel to ac to meet the axis minor in m. Join mR, and then mRG is the angle ψ between the tangent to the ellipse at p and the generating line of the cylinder passing through p: for

$$\tan GRm = \cot ORm = \frac{OR}{Om} = \frac{Ol\tan\Omega}{Om} = \frac{r}{Om}\tan\Omega = \tan\psi,$$

$$\therefore \angle\, GRm = \psi \text{ or } (\pi - \psi).$$

49. From R draw RH, making with RG an angle ϕ equal the angle of the intrados. Then mRH is the angle between the tangents to the ellipse and intradosal spiral coursing joint at the point p.

50. In the case where the right sector of the arch is less than a semicircle, the elliptic arc ac must be produced so as to complete the semi-ellipse $a'lc'$, Fig. 67. Make, as before, $OlR = \Omega$, and from R draw RG at right angles to $a'c'$. From p draw pm parallel to $a'c'$ to meet the axis minor in m. Join mR, and then $mRG = \psi =$ the angle between the generating line passing through p and the tangent to the ellipse at p.

51. If $HRG = \phi = \angle$ of the intrados, then mRH and its supplement are, as before, the angles between the tangents

to the ellipse and to the intradosal coursing joint at the point p.

52. It is evident that the angle GRm for two points p, p' in the *same horizontal* plane remains the same. This property is very useful in the determination of the angles of the quoins of the arch, such as the angles $\gamma a_6 a_5$, $a_6 a_5 a$, in Fig. 89.

It may be useful to remark, that if from l as a centre, Fig. 63, with radius Oc, we describe an arc cutting Oc in s and join ls, then $\angle Osl = \Omega$, Art. 46. If then we draw lR at right angles to sl, we shall at the same time make $lRO = 90° - \Omega$, and $\therefore$ also $RlO = \Omega$.

Let the developments, Figs. 8 and 9, 20 and 21, be placed as in Art. 18, γa_{17} and aa_{16} being joints on the intrados corresponding to $\gamma'\beta_{17}$ and $a'\beta_{16}$ on the extrados. $a\beta\delta\gamma$ is the form of an arch-stone on the intrados. From the points a, β in aa_{16} draw lines aa', $\beta\beta'$ at right angles to lm or LM, cutting $a'\beta_{16}$ in a', β'. Then a', β' are the two corners of the arch-stone on the extrados corresponding to a, β on the intrados, and in the same way the two remaining corners γ', δ' may be found. If p be *any* position of p_0, Art. 19, on the intrados, and if it be required to find the corresponding position of P_0, draw pr parallel to ha meeting lm in r, and through r, p, draw rr', pP at right angles to lm or LM; rr' meeting LM in r'. Draw $r'P$ parallel to $A'H$ meeting pP in P; then it is manifest that P is the point sought.

◆

CHAPTER III.

53. If tangents be drawn to the intradosal and extradosal helices at contemporaneous positions of p_0 and P_0, Fig. 9*, these tangents will lie in parallel tangent planes, and they will also meet the generating lines (along which the planes touch the cylinders) at angles ϕ and Φ respectively. Hence, if through any point we draw straight lines parallel to the tangents at p_0, P_0, the angle between them will be $= \Phi - \phi$.

54. In the construction of the oblique bridge with spiral courses, the twist of the beds of the arch-stones is so slight that the following process gives a sufficiently near approximation to the proper form. Two pieces of thin straight board ac, $a'c'$, Fig. 78, must be provided. The opposite edges, ad, bc, of one of these strips, as ac, are parallel. Also let $a''c'$ be parallel to $a'd'$, and make the angle $b'c'a'' = \Phi - \phi =$ difference of the angles of extrados and intrados; or, in other words, the opposite sides $a'd'$, $b'c'$ of $a'c'$ are to be inclined at an angle $\Phi - \phi$.

55. Let the arch-stone to be formed, Fig. 80, be placed with its bed $\alpha\beta\gamma\delta$ horizontal. Channels called "chisel drafts" must be cut in directions $\alpha'\delta'$, $\beta'\gamma'$, so as to be *parallel* and at a distance $R - r = \alpha\delta = \beta\gamma$ apart. The bottoms of these chisel drafts must be carefully formed so that they may be everywhere in contact with the applied straight-edge. Also the depths must be so adjusted that when the "winding strips" $a'c'$, ac, are applied along $\alpha'\delta'$, $\beta'\gamma'$ respectively, with their planes vertical, the edges not in contact with the stone, $a'd'$, ad, may be "out of winding," or in one plane. The superfluous stone must be removed, and the proper form will be attained when a straight-edge applied transversely, or in

directions approximately parallel to $\alpha\delta$, $\beta\gamma$, are everywhere in contact with the stone and touch the *bottoms* of the chisel drafts $\alpha'\delta'$, $\beta'\gamma'$.

56. The winding strips are frequently applied in the directions $\alpha\delta$, $\beta\gamma$, Fig. 80. In Fig. 81, let $e'j'$, ej', be the winding strips suited to this case. $fj' = f'j'' = R - r$. Produce $\alpha\delta$, $\beta\gamma$ to cut $a'c'$ in $f''e''$, lk. Then $e'f'$, Fig. 81, must be made as much greater than $i''j''$ as $f''e''$ is greater than kl in Fig. 80. The *parallel* chisel drafts $\alpha\delta$, $\beta\gamma$ must be cut at the distance kf'', so that when the winding strips are inserted, their upper edges may appear to be in one plane.

57. Make the angle $qOr = \Phi - \phi$, Fig. 77, and with centre O and radius $d''a' = l$, Fig. 78, describe the arc qr, then qr is approximately the excess of $b'a'$ above $c'd''$. In this case bd, $b'd''$ must be applied *parallel* at a distance $R - r$, and in the direction of the *length* of the arch-stone. Again, with centre O, Fig. 67, and radius $\alpha\beta = \gamma\delta$, Fig. 80, describe the arc st, then st is very nearly the excess of $e'f'$ above $i''j''$, Fig. 81, $R - r$ being $= ei = f'j'' =$ the length of the winding strip. $e'j''$, ej' must be applied along $\alpha\delta$, $\beta\gamma$, respectively, so as to be parallel and at a distance Os, Fig. 77.

58. Mr. Buck gives particular directions for the *divergency* of the chisel drafts $\alpha\delta$, $\beta\gamma$, whilst the mode in which he determines the excess of $e'f'$ above $i'j'$ gives practically the above result. Mr. Buck makes $\gamma\delta : \alpha\beta :: A'H : ah$, Figs. 8 and 9, 20 and 21. It is quite true that, when the arch-stone is properly formed, as in Fig. 87,

$$ab : \alpha\beta :: A'H \text{ (Figs. 9 and 21)} : ah \text{ (Figs. 8 and 20)}:$$

but it ought to be recollected that the mere lengths of ab, $\alpha\beta$ of the edges of the *finished stone* need not be considered in finding the dimensions of the winding strips. As Mr. Buck so distinctly directs the divergence to be carefully observed, I refer to p. 147 of vol. XII. of the *Civil Engineer and Architect's Journal*, for a full statement of my reasons for objecting to Mr. Buck's directions. I suppose a plane to pass through three corners of a *perfectly* shaped stone, and then from the fourth corner I let fall a perpendicular on this plane. On applying this formula to Mr. Buck's example II. (page 27), I find the length of the perpendicular equals 3.408 inches ($\alpha\beta$ being 42 inches, and $\gamma\delta$, 47 inches) instead of 3 inches. If $\alpha\beta = \gamma\delta = 42$ inches, then I allow that $e'f' - i'j'' = 3$ inches, Fig. 81. But if we suppose $\delta\gamma$ to be 47 inches, we must also increase $e'f'$, so that

$$e'f' - i'j'' = 3 \times \tfrac{47}{42} = 3\left(1 + \tfrac{5}{42}\right) = 3 + \tfrac{15}{42} = 3 + .36 = 3.36 \text{ inches.}$$

Example II. is an extreme case, and the difference of the results (I believe I may say the error of Mr. Buck) would generally be much less. By applying the winding strips $\alpha\delta$, $\beta\gamma$ so as to be parallel, the trouble of calculating the divergence is avoided, and a more correct result is obtained.

59. In all the plates we have supposed the oblique bridge to be *left*-handed; that is, if a person be approaching the bridge, Fig. 4, in the direction *FG*, perpendicular to the face of the arch, he must turn to the *left* to pass along the axis of the arch. The intradosal and extradosal coursing joints of such an arch are *right-handed* helices. If a person place himself at the intrados of an arch-stone, Fig. 80, and look towards the stone, the *left*-hand corner of the extrados (δ) is the one that must be depressed.

60. Having completed the formation of one side of the arch-stone, we must next proceed to shew how the remaining surfaces may be found from this one. It is evident from the mode in which the twisted bed of the stone has been supposed to be formed, that in the neighbourhood of $\alpha\beta$, Fig. 80, the finished surface does not depart sensibly from a plane, for $b'a''$ is always very small compared with $a'd'$, Fig. 78. The intersection of this plane and the soffit of the bridge will approximately be an arc of an ellipse at the extremity of the minor axis. Draw tf a tangent to the projection of the intradosal coursing joint at t, Fig. 16. This will cut the axis of the cylinder OO' at an angle ϕ, and the semi-axis major of the elliptic section formed by the intersection of a vertical plane through tf and the cylinder,

will be $tf = r \operatorname{cosec} \phi$. Hence the radius of curvature at the extremity of the axis minor

$$= \frac{(\tfrac{1}{2}\text{ axis major})^2}{\tfrac{1}{2}\text{ axis minor}} = \frac{(r \operatorname{cosec} \phi)^2}{r} = r \operatorname{cosec}^2 \phi.$$

61. Take $AB = r$, Fig. 82; make the angle $BAC = (90° - \phi)$, and draw BC at right angles to AB, intersecting AC in C. At C draw CD at right angles to AC, meeting AB produced in D. Then DA is the required radius for

$$DA = \frac{DA}{AC} AC = \frac{DA}{AC} \times \frac{AC}{AB} \times AB,$$

$$= (\operatorname{cosec} \phi)(\operatorname{cosec} \phi) r = r \operatorname{cosec}^2 \phi.$$

62. Provide a template, Fig. 83, such that the concave edge BC may be an arc of a circle to a radius DA, Fig. 82, or $= r \operatorname{cosec}^2 \phi$. Now apply the template, Fig. 83, to the worked bed of the stone, Fig. 80, so that $\alpha\beta$ may be parallel to the *tangent to the middle point* of the arc BC made use of in that particular case, and then mark the stone along BC. This is the curve in which the soffit intersects the bed of the arch-stone. Also from this curve $\alpha\beta$, Fig. 86, we must draw βb, $a\alpha$, normals to the curve $\alpha\beta$ at the points $\alpha\beta$ respectively, which will give the proper excess of ab above $\alpha\beta$. Thus we have determined $a\alpha$, $\alpha\beta$, βb, and no particular care is needed for the extrados ab.

63. The surface of the intrados is a cylinder of radius r. The intradosal edge $\alpha\beta$ cuts all the generating lines of this cylinder at an angle ϕ. Provide an arch-square, Fig. 95, the interior edge of fg being an arc of a circle of radius r, and the straight-edge ef being directed towards the centre of the arc fg. Fig. 86 is a perspective view of this arch-square applied to an arch-stone, and Figs. 91 and 92 are projections of the same. The stock fe must be applied in the direction of a normal to the curve $\alpha\beta$, drawn on the bed of the stone, Fig. 92; and fg must be applied so as to cut all the generating lines of the soffit at right angles, or it must be inclined at an angle $(90° - \phi)$ to $\alpha\beta$, $\gamma\delta$, Figs. 86 and 91. The stock fe may be rigidly attached to a guide, which, resting on the bed

$\alpha\beta ba$ of the stone, will give *fg* the required inclination, Fig. 91; and also at the same time, by being placed in contact with the curve $\alpha\beta$, give the right direction to the stock *fe*, Fig. 92. The soffit $\alpha\delta$ must be completed, so that wherever the arch-square, Fig. 95, is applied in the manner above directed, the angle *f* may always be in contact with $\alpha\beta$, and *fg* with the surface $\beta\gamma$. Two or three channels may be cut in the soffit to receive *fg* to the proper depth, and then the whole surface $\alpha\delta$ may be completed by removing the stone, so that a straight-edge, applied in the same direction as the lines used in shading figures 86 and 91, making an angle ϕ with $\alpha\beta$ or $\gamma\delta$, may be in contact with the bottoms of the chisel drafts and with the stone.

64. Also it is evident, that as the cross-joints $\alpha\gamma$, $\beta\delta$ are supposed to be drawn at right angles to $\alpha\beta$, $\gamma\delta$ in the development, $\beta\delta$ may be considered to be an arc of an ellipse at the extremity of the axis of an ellipse. The semiaxes of this ellipse will be r, and a straight line drawn from *t* at right angles to *tf* in Fig. 16, to meet *hd*. The radius of curvature at this place will now equal

$$r \operatorname{cosec}^2\left(\frac{\pi}{2} - \phi\right) = r \sec^2 \phi. \quad \text{Art. 60.}$$

Make $AB = r$, Fig. 82, and from A draw AC' inclined at an angle ϕ to AB, and draw BC' at right angles to AD, meeting AC' in C'. From C' draw $C'B'$ at right angles to AC', meeting AB produced in B'. Then AB' is the required radius; for

$$AB' = \frac{AB'}{AC'} \times \frac{AC'}{AB} \times AB = (\sec \phi)(\sec \phi) \times r = r \sec^2 \phi.$$

And it will be sufficiently accurate to consider the small arc $\beta\delta$, Fig. 86, to be a small arc of a circle of radius $r \sec^2 \phi$. Hence, if the arch-square be formed so that the edge *fg*, Fig. 95, is an arc of a circle of radius $r \sec^2 \phi = AB'$, Fig. 82, and if *ef* point towards the centre, then the stock *fe* may be applied in the direction of the normal to the curve $\alpha\beta$ on the bed of the stone, as has been before directed, but in this case *fg* must be parallel to $\beta\delta$. The direction in which the straight-edge must be applied in giving the true cylindrical surface to the soffit $a\delta$ of the stone, Fig. 76, will be very easily obtained by providing a template with an angle $= \phi =$ *dha*, Figs. 8 and 20; and then one edge being applied along $a\beta$, the other edge will be in the directions of the generating lines of the cylindrical surface.

65. Provide a rectangular piece of sheet-iron or other flexible substance, the shorter sides of which are exactly equal the distance of consecutive coursing joints in the development of the soffit, Figs. 8 and 20. When this is pressed on the properly formed surface $a\delta$, Fig. 86, so that one of the longer sides may coincide with $a\beta$, then the other sides will determine $a\gamma$, $\gamma\delta$, and $\delta\beta$. If the template be either too long or too short for a particular stone, it may be easily placed so as to give $a\gamma$, $\beta\delta$. The line $\gamma\delta$ is to be used in working the remaining bed of the stone. In ordinary cases the common square may be applied to determine γ and δ with sufficient accuracy.

66. The thickness *bd*, *ac* of the arch-stones at the extrados is known from Figs. 9 or 21. Measure the thickness *bd*, *ac*, and connect *cd* by a "chisel draft." Then for the completion of the lower bed nothing more is necessary than to remove the superfluous stone till a straight-edge, applied in the direction of the normal at each point of the curve $\gamma\delta$, may also pass through $\gamma\delta$ and *cd*, and be everywhere in contact with the surface of the stone. The lines δd, γc may be determined from $\gamma\delta$ just in the same way as *aa*, βb were found from $a\beta$.

67. The two ends are now the only surfaces of the arch-stone that remain to be completed, as it is not necessary to take great care respecting the form of the extrados *adcb*. The proper twist of βa will have been obtained when a straight-edge *qr* is always in contact with the surface of the stone $b\beta\delta d$ when $\beta r = \delta q$, Fig. 86. All the arch-stones except the quoins are formed in the above manner.

68. In preparing the quoins the two beds $\alpha\beta ba$, $\gamma\delta dc$, the soffit $\alpha\beta\delta\gamma$ and one of the ends $\alpha ac\gamma$ must be formed in the same manner as for an ordinary arch-stone. The remaining end is a plane, and remains to be determined. Suppose that we require to form the quoin $a_{21}b_{22}$, Fig. 49. From the development Fig. 8, we may take the distance from a_{22} to the first heading joint $= \gamma a_{22}$ in Fig. 88, which determines a_{22}, one of the points in the face of the arch. Again, the $\angle\ \gamma a_{22}a_{21}$, Fig. 88, may be found either from the development, Fig. 8, as is commonly done, or more accurately in the manner described in Arts. 47–52, Figs. 63 and 67. Having set the bevil, Fig. 94, to this angle, that instrument must be applied as in Fig. 87, and thus a *second* point a_{21} in the required plane face will be found.

69. Let LM and lm be coincident in Fig. 56, and let Lpc, LPC be half the developments of the intrados and extrados. Suppose the lines ap, aP to be drawn inclined to LM at angles ϕ and Φ respectively, then these two lines will be corresponding joints on the intrados and extrados. Suppose it to be the joint $a_{22}a_{22}$, Fig. 8. From the point p, where ap meets Lpc, draw pn parallel to LE cutting aP or aP produced in n. Then when the point p_0 in Fig. 9* has come to the face of the arch, the point P_0 has to describe a space nP before it arrives at the same plane.

70. If now through the previously determined point a_{22}, Fig. 88, we draw $a_{22}n_{22}$ a normal to the curve γa on the bed of the stone γd, and if along the extradosal helix cd we measure $n_{22}b_{22} = nP$ in Fig. 56, the point b_{22} will be a *third* point in the required plane face, for a_{22} and n_{22} are respectively the contemporaneous positions of p_0 and P_0 in Fig. 9*. As a test of the accuracy of the preceding determinations a *fourth* point b_{21} in the plane face $a_{22}b_{22}b_{21}a_{21}$ may be found.

71. The joint a_{21} corresponds to a_6, and the length of a_6n_6 would have been found to be equal $a_{21}b_{21}$ exactly. So also $a_1n_1 = a_{26}n_{26}$, $a_2n_2 = a_{25}n_{25}$ &c. But in the formation of the quoins, care must be taken to measure these distances in the right directions. Fig. 88 is a view of the quoin $a_{21}b_{22}$, where

it is seen that $n_{22}b_{22}$ is measured in the opposite direction from the normal compared with b_6n_6, Fig. 89. The rule is obvious— for the half of the face of the arch on the side of the *obtuse* corner of the abutment *cC,* all the angles $\gamma a_{22}b_{22}$, $aa_{21}b_{21}$ &c. are *obtuse* angles, and those next the *acute* angle of the abutment are acute angles at the intrados, as γa_6b_6, aa_5b_5 &c. In some cases, long arch-stones have been completed, and then divided into two such stones, as Figs. 88 and 89, by means of a saw.

72. The most convenient mode in which instructions respecting the forms of the quoins can be conveyed to workmen is as follows. Draw lines parallel to *ac,* Figs. 63 or 67, through the *middle* points of aa_1, a_1a_2, a_2a_3 &c. of the edges of the quoins meeting the vertical semi-axis in m_1, m_2 &c., and join Rm_1, Rm_2 &c. Provide a board *BCD* having a straight-edge *BAC,* Fig. 93. From *A* draw lines making the same angle with *BA,* that Rm_1, Rm_2 &c. make with *RH.* These may be numbered according to the joint. Thus for the angle HRm_1, we place 0 and 1 on the two sides of the line drawn in Fig. 93. So also for the angle ARm_2 we place the figures 1 and 2 &c. It will be found sufficient to determine these angles for one half of the ellipse. The numbers for both sides may be inserted as in Fig. 93.* The stock *tr* of the bevil, Fig. 94, can be applied to the straight-edge *BAC,* and from the known number of the joint the blade *qs* can be set to the proper angle. The bevil must then be applied to the point a_{22} as in Fig. 87, and thus a_{21} will be found. If a_{21} be known, then *tr* must be applied to $a\beta$, and *rq* will then give the point a_{22}. The angle taken from Fig. 93 is for the middle point between a_{21} and a_{22}.

73. Similar contrivances are shewn in Figs. 84 and 85, for determining the angles on the beds γa_6b_6, aa_5b_5, &c. *BAC* is an arc of a circle whose radius is equal *DA,* Fig. 82,

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* In Fig. 93 the figures 24, 25, 26, and 27 have been slightly misplaced; 24 ought to occupy the place of 25, 25 that of 26, and 26 that of 27. It will be seen in Fig. 59, that a_1 and a_{26}, a_2 and a_{25}, a_3 and a_{24}, &c. are two and two in the same horizontal plane.

Art. 60, = also r cosec$^2 \phi$. DA points towards the centre, and $DA = R - r$ = the thickness of the arch. The radius of the circle passing through D is equal R cosec$^2 \Phi$, and may be determined by construction precisely as r cosec$^2 \phi$ was found in Fig. 82, R and Φ being used instead of r and ϕ. The lengths of Pn, Fig. 56, for each course for one half of the development must be found, and these distances must be set off from D on the arc of the circle passing through D, and these points must be properly numbered, as in Fig. 84. Suppose now a workman to have prepared a quoin, Fig. 90. The two points a_6, a_5 having been found, as in Fig. 87. The stock tv of the bevil, Fig. 85, is applied to the curve BAC, Fig. 84. The blade uw is made to coincide with $A6$, and is there clamped. The bevil is then applied to the stone, as in Fig. 90, the edge of uw passing through a_6, and then the intersection b_6 of the same edge with the extradosal helix cd will be a *third* point in the plane $a_5a_6b_6$ required.

74. Another mode of obtaining the angles $\gamma a_6 b_6$, aa_5b_5, &c. may be here noticed. Find the focus Q to which the joints of the arch-stones on the face of the arch converge. Divide the elevation of the intrados alc, as in Figs. 16, 17, and 24, or by any other legitimate method, so that the points $a_1a_2a_3$ &c. in the elevation may correspond to the points $a_1a_2a_3$ &c. in the developments, Figs. 8 or 20. Let a'', Figs. 14 or 26, be one of these points, as a_{22}. Join Qa'', and produce it to meet the ellipse ALC, in which the face of the arch cuts the extrados, in b''. Then $a''b''$ is the length of $a_{22}b_{22}$ in Fig. 88. Hence, if with centre a_{22} and radius = $a''b''$, Figs. 14 or 26, we describe an arc cutting the helix cd in b_{22}, it is evident that b_{22} is a point sought in the face of the arch. This method may be used for the formation of the instrument, Fig. 84.

It is well known that $a_{22}b_{22}$ or $a''b''$ is in reality a curve, but $a_{22}b_{22}$ are two points in the helices of the intrados and extrados, and $a''b''$ gives correctly the distance of these two points measured in a straight line, and the distance $a_{22}b_{22}$ is also supposed to be measured in a straight line in Fig. 88.

The methods of forming the quoins, as recommended above, are simple, and the degree of accuracy in the results will depend entirely on the nicety of the working drawings, which must be made to a scale commensurate with the degree of accuracy required.

The quoins of arches are generally formed with steps, as $b_1b_2b_3b_4$ &c., Figs. 49, 51, and 53; but we have always supposed the outline to be elliptic, like *AL*, Fig. 53. If the beds be accurately formed for such a case, these surfaces may be easily produced to give the forms of Figs. 49 and 51.

75. The intradosal courses of the oblique bridge meet the impost at an angle ϕ equal the angle of the intrados. It is necessary that the imposts of the abutments should be properly formed to receive the ends of the courses. The shaded parts of the developments along *ab*, *AB*, Figs. 8 and 9, are in reality parts of the imposts. Fig. 64 is an isometrical projection of an impost of an oblique bridge where $\lambda\mu\nu$, $\lambda'\mu'\nu'$ are the same triangles as $\lambda\mu\nu$, Fig. 8, and $\lambda'\mu'\nu'$, Fig. 9, respectively, and they are so placed that the straight lines $\lambda'\lambda$, $\nu'\nu$, $\mu'\mu$ &c. being produced would intersect the axis of the cylinder at right angles.

76. Now $\nu\lambda\mu = \phi$, and $\lambda\nu\mu =$ a right angle in the development. Hence if upon *ab*, Fig. 68, the actual length of the check $\lambda\mu$, we describe the semicircle *acb*, and from *a* draw *ac* inclined at an angle ϕ to *ab*, meeting the circle in *c*, and join *bc*, then *bca* will be a right angle, and the triangle *acb* will be the proper form of the template for working the intrados of the checks $\lambda\mu\nu$, Fig. 64. Also, from Fig. 9, it is known that $\angle \nu'\lambda'\mu' = \Phi =$ the angle of the extrados. At the point *a* in *ba* make the $\angle bac' = \Phi$. Let *ac'* cut the perpendicular *cn* let fall from *c* on *ab*, when produced backwards, in the point *c'*. Join *c'b*, and then,

since $\dfrac{r}{R} = \dfrac{\tan\phi}{\tan\Phi} = \dfrac{cn}{c'n}$, the $\triangle abc'$ will be the form of the template for the extrados $\lambda'\mu'\nu'$, Fig. 64.

PART II.

ON THE CONSTRUCTION OF THE OBLIQUE BRIDGE WITH EQUILIBRATED COURSES.

77. IN Figs. 8 and 9, the coursing joints drawn on the developments are straight lines, but we no longer suppose them to be such. The coursing joint of the intrados of an obliquilibrated oblique bridge intersect at right angles the curve formed by the intersection of the soffit, with any plane parallel to the faces of the arch. Sometimes awkward and expensive substitutes for a proper oblique bridge are obtained by building a series of thin independent direct arches, as in the plan Fig. 4. If we suppose the number of these arches very much increased in number, and their widths diminished, we shall approximate to the equilibrated bridge.

78. It has been shewn (Art. 39) that the equations to the heading-spiral *atc*, Fig. 29, are

$$\left.\begin{aligned} y &= r\theta \\ x &= -r\cot\Omega\sin\theta \end{aligned}\right\} \ldots\ldots\ldots\ldots (1).$$

Hence the equation to $c'a'$, the development of the curve formed by the intersection of a plane parallel to, and at a distance $d = tt'$ from the face of the arch, will be

$$\left.\begin{aligned} y &= r\theta \\ x &= -r\cot\Omega\sin\theta - d \end{aligned}\right\} \ldots\ldots\ldots\ldots (2).$$

Assume that $Y = f(X)$ (3) is the equation to a curve which intersects (2) at right angles in the point x, y. Then at this point we must have $y = Y$, $x = X$, and $\frac{dy}{dx} = -\frac{dX}{dY}$,

or
$$\frac{r}{-r\cot\Omega\cos\frac{Y}{r}} = -\frac{dX}{dY};$$

therefore
$$\frac{dX}{dY} = \frac{\tan\Omega}{\cos\frac{Y}{r}},$$

or
$$X + C = -r\tan\Omega\int\frac{\sec^2\left(\frac{\pi}{4}-\frac{Y}{2r}\right)}{\tan\left(\frac{\pi}{4}-\frac{Y}{2r}\right)}\,d\left(\frac{\pi}{4}-\frac{Y}{2r}\right)$$
$$= -r\tan\Omega\log_e\tan\left(\frac{\pi}{4}-\frac{Y}{2r}\right)\ldots\ldots(4),$$

which is the equation to the required family of curves containing one arbitrary constant C, and independent of d in (2).

79. Suppose the curve to pass through the origin, then when $X = 0$, $Y = 0$, and
$$C = -r\tan\Omega\log_e\tan\frac{\pi}{4} = 0.$$

Hence the equation to the curve a_1ta_2 is
$$X = -r\tan\Omega\log_e\tan\left(\frac{\pi}{4}-\frac{y}{2r}\right)\ldots\ldots(5);$$

or writing x and y for X and Y, and assuming as before $y = r\theta$, we get
$$\left.\begin{aligned} y &= r\theta \\ x &= -r\tan\Omega\log_e\tan\left(\frac{\pi}{4}-\frac{\theta}{2}\right)\end{aligned}\right\}\ldots\ldots(6),$$

for the equations to the coursing joint a_1ta_2.

80. Let the development, Fig. 29, be supposed to be wrapped round a cylinder of radius r, tm conciding with that generating line of the cylinder which is vertically above the axis of the cylinder. Take now the axis of the cylinder for the axis of y, a vertical line passing through t, Fig. 29, for the axis of z, and the axis of x at right angles to the

axis of y, and in the same horizontal plane with it. Suppose that in Fig. 29, $mu_6 = r\theta$ and $tm = -r \tan \Omega \log_e \tan\left(\frac{\pi}{4} - \frac{\theta}{2}\right)$. Let xyz be the coordinates of the point m when Fig. 29 assumes its proper cylindrical form; then

$$\left.\begin{aligned} y &= tm = -r \tan \Omega \log_e \tan\left(\frac{\pi}{4} - \frac{\theta}{2}\right) \\ x &= -r \sin \frac{mu_6}{r} = -r \sin \frac{r\theta}{r} = -r \sin \theta \\ z &= r \cos \frac{mu_6}{r} = r \cos \frac{r\theta}{r} = r \cos \theta \end{aligned}\right\} \ldots . (7).$$

Hence, if in equations (6) we give to θ any value, we shall find xy, the coordinates of some point m' in the development, Fig. 29, and if we give to θ the *same* value in the expressions (7), we shall obtain the values of the coordinates of the *same* point m', when Fig. 29 is wrapped round a cylinder of radius r.

81. It is evident that if we combine the three equations (7) in pairs, we shall obtain the equations to the projections of that curve on the several coordinate planes. Thus

$$x = -r \sin \theta,\ y = -r \tan \Omega \log_e \tan\left(\frac{\pi}{4} - \frac{\theta}{2}\right) \ldots . (8),$$

are the equations to $o_8 o_8'$, Fig. 30, the projection on the horizontal plane xy:

$$y = -r \tan \Omega \log_e \tan\left(\frac{\pi}{4} - \frac{\theta}{2}\right),\ z = r \cos \theta \ldots\ldots (9),$$

are the equations to $r'r$, Fig. 31, the projection of the coursing joints on the plane yz, or on a plane parallel to the abutments.

82. The projection of the coursing joint on the plane xz will manifestly be a circle. The equation to $s_8 s_0 s_6'$, the projection of the same curve on a vertical plane which cuts the horizontal plane xy in WV, may be found by supposing

the axes of x and y in (7) to be turned through an angle $x t' V = \beta$ about the axis of z. Thus, let xyz be the coordinates of a point referred to the old axes, $x'y'z'$ those of the same point referred to the new axes.

Then

$$\left.\begin{aligned} x' &= x \cos\beta + y \sin\beta \\ &= -r\cos\beta\sin\theta - r\tan\Omega\sin\beta\log_e\tan\left(\frac{\pi}{4}-\frac{\theta}{2}\right) \\ y' &= -x\sin\beta + y\cos\beta \\ &= r\sin\theta\sin\beta - r\tan\Omega\cos\beta\log_e\tan\left(\frac{\pi}{4}-\frac{\theta}{2}\right) \\ z' &= z = r\cos\theta \quad , \end{aligned}\right\} \ldots.(10).$$

If in (10) $\beta = 90° - \Omega$, we have, for the equations to the coursing joints that appear in the elevation, Fig. 53,

$$\left.\begin{aligned} x' &= -r\sin\Omega\sin\theta - r\sin\Omega\log_e\tan\left(\frac{\pi}{4}-\frac{\theta}{2}\right) \\ z' &= r\cos\theta \end{aligned}\right\}$$

83. The forms of the beds of the arch-stones must be determined before we can find the equations to the coursing joints of the extrados. Suppose that A_0C_0, in Fig. 9*, be caused to move in direction of its length which coincides with the common axis of the intrados and extrados. Let the arm B_0P_0 turn as A_0C_0 advances, so that p_0 may describe the intradosal coursing joint (7), then P_0 will trace out that of the extrados. The equations of the path to p_0 are

$$x = -r\sin\theta,\quad y = -r\tan\Omega\log_e\tan\left(\frac{\pi}{4}-\frac{\theta}{2}\right),\quad z = r\cos\theta$$

hence manifestly the equations to the path of P_0 will be

$$X = -R\sin\theta,\quad Y = -r\tan\Omega\log_e\tan\left(\frac{\pi}{4}-\frac{\theta}{2}\right),\quad Z = R\cos\theta \quad \ldots\ldots..(11),$$

The equations to the projection of curve (11) on the plane (XY) are

$$X = -R\sin\theta,\quad Y = -r\tan\Omega\log_e\tan\left(\frac{\pi}{4}-\frac{\theta}{2}\right),$$

and on the plane (YZ), which is parallel to the plane of the abutments,

$$Y = -r \tan \Omega \log_e \tan\left(\frac{\pi}{4} - \frac{\theta}{2}\right), \ Z = R \cos \theta.$$

84. It is also evident that if the extrados be developed, the curve (11) will have for its equations

$$Y = R\theta, \quad X = -r \tan \Omega \log_e \tan\left(\frac{\pi}{4} - \frac{\theta}{2}\right) \dots (12),$$

which is A_1tA_2, in Fig. 29, where ltm is the axis of X.

85. Comparing equations (6) and (12), we find that for a given value of θ, $\frac{Y}{y} = \frac{R}{r}$ and $X = x$. Hence if, in Fig. 29, we draw any line mu_6j parallel to the axis of y or at right angles to xx', and make $mj : mu_6 :: R : r$, the point j will be a point in the coursing joint A_1tA_2.

86. In the developments of the intrados and extrados, Figs. 34 and 40, we have $ba = lm = dc = BA = LM = DC$. If the equations to any *intradosal* coursing joint $\mu_{16}a_{16}$ be

$$y = r\theta, \quad x = -r \tan \Omega \log_e \tan\left(\frac{\pi}{4} - \frac{\theta}{2}\right) - d,$$

m being the origin of coordinates, then the equations to the *corresponding* extradosal coursing point $\nu_{16}b_{16}$, Fig 40, are

$$y = R\theta, \quad x = -r \tan \Omega \tan\left(\frac{\pi}{4} - \frac{\theta}{2}\right) - d,$$

where M is the origin, $M\nu_{16} = d = m\mu_{16}$.

87. The preceding formulæ would be of little practical use if they did not admit of considerable simplification. The scales in Figs. 98, 99, and 100 have been devised for this purpose. Fig. 97, an ordinary scale of equal parts, is used in dividing the scale, Fig 98, which is to be used for finding the values of $-r \tan \Omega \log_e \tan\left(\frac{\pi}{4} - \frac{\theta}{2}\right)$. The reason why the graduations 10°, 20°, 30°, &c., in Fig. 98, are so placed is as follows. θ has certain values 10°, 20°, 30°, &c. given to it, and from tables of log-tangents the numerical values

of L_{10} tan of $(45 - 5^{\circ}), (45^{\circ} - 10^{\circ}), (45^{\circ} - 15^{\circ})$, &c., are found, thus:

values of θ	0°	10°	20°	30°	40°	&c.
values of $\left(45^{\circ} - \frac{\theta}{2}\right)$	45°	40°	35°	30°	25°	&c.
values of L_{10} tan $\left(45^{\circ} - \frac{\theta}{2}\right)$	0	.0761	.1548	.2386	.3313	&c.

Thus 0° on (*E*) is placed opposite 0 on the scale of equal parts (*B*), 10° on (*E*) is placed opposite .0761 on (*B*), 20° on (*E*) is placed opposite .1548 on (*B*), and so on. Fig. 99 is a scale of natural sines, and Fig. 100 is a scale of cosines or of versed sines to a radius 100, Fig. 97.

88. It appears that the equations to the development of the intradosal heading spiral *ala'*, Fig. 6, are $y = r\theta$, $x = -r \cot \Omega \sin \theta$. In Fig. 61, draw *Oa*, *Ol* at right angles to each other. Make *Ol* = the arc *al'*, Fig. 3, = half of the length of the right section of the intrados of the arch. Also take *Oa* equal the half of *ec*, Fig. 4. The right section is in this case a semicircle, or $a = 90^{\circ}$. Place the zero of any convenient scale of equal parts *BC* at *l*, Fig. 61, and turn the scale about that point till the 90th division is made to fall upon *Oa* or *Oa* produced. Through any points, as 10, 20, 30, &c. draw lines parallel to *Oa*. Then apply a scale of natural sines *FG*, as in Fig. 62; 90° falling on *the point a*, and the zero on *the line Ol*. Then through the divisions 10, 20, 30, &c., *numbered the same* as those used in Fig. 61, draw lines parallel to *Ol*. The intersections of these lines, as indicated in Fig. 62, will determine points in the development of the intrados.

89. The same method may be applied to cases where the right section of the arch is a segment less than a semicircle. As before, in Fig. 65, make *Ol* = *al'*, Fig. 18, = an arc subtending an angle $a = (53^{\circ}\ 8')$ in this example, and *Oa* = half of *ec*, Fig. 4. The zero of *BC*, a scale of equal parts, is made to coincide with the *point l*, and 53° 8' or 53.13, to fall on the *line Oa*. Again, the scale of sines *FG* is

applied so that 0° is on the *line* Ol, and 53° 8′ on the *point* a. The lines must be drawn to intersect as before. This is the most convenient method of determining the edges of the developments of the cylindrical surfaces of oblique arches.

90. The equations to the coursing joint a_1ta_2 in Fig. 29, are $y = r\theta$, $x' = -r \tan \Omega \log_e \tan \left(\frac{\pi}{4} - \frac{\theta}{2}\right)$. The several values of y will be found precisely as in Figs. 61, and 65. We also find that, if $\theta = 49° \ 26' \ 18''$, $\log_e \tan \left(\frac{\pi}{4} - \frac{\theta}{2}\right)$ is $= -1$, and therefore $x = r \tan \Omega$. Also, if $\theta = 74° \ 35' \ 8''$, then $\log_e \tan \left(\frac{\pi}{4} - \frac{\theta}{2}\right) = -2$, and consequently the corresponding value of x is $= 2r \tan \Omega$. B_1 is placed at 49° 36′ 18″, and B_2 at 74° 35′ 8″, in Fig. 98, which are two very important points. B_1 falls opposite .43429 on BC, and B_2 falls opposite .86859 on the same scale, Figs. 97 and 98. The corresponding angles are marked C_1 and C_2 on the scale of sines FG, Fig. 90, and D_1 and D_2 in the scale of cosines, Fig. 100.

91. In Fig. 29, make $ty' =$ arc al', Fig. 3, and apply the scale of equal parts, as in Fig. 61, and thus determine the lines v_1u_1', v_2u_2' &c. Also βv must be ruled at the division 74° 35′ 8″, or at 74.585. In the plan, Fig. 29*, (which is the same as Fig. 4), take $cG = 2r =$ twice the radius of the intrados, and draw GG' at right angles to cG, meeting ca or ca produced in G'. From v measure along $v\beta'$, Fig. 29, a distance $v\sigma = G'G$, Fig. 29*. Apply the scale ED, Fig. 98, so that zero may be on the *line* $y'y$, and B_2 on the *point* σ. Then draw lines parallel to yy' through the divisions numbered the *same* as those used in the determination of the lines v_1u_1', v_2u_2' &c., as in Fig. 61. In the Fig. 29, these are 10°, 20°, 30° &c. And thus the points $u_1'u_2'u_3'$ &c. in the coursing joint will be found.

92. If the line aa' had been ruled at the division 49° 36′ 18″, or at 49.6 on the scale applied as in Fig. 61, we must then have taken $cg = r =$ the radius of the intrados,

and drawn gg' at right angles to hc meeting ac or ac produced in g', Fig. 29*, then $gg' = r \tan \Omega$. In Fig. 29 make $n\rho' = gg'$, Fig. 29*, and adjust the scale ED, Fig. 98, so that zero may be on the *line* yy', and B_1 on the *point* ρ'. Lines may be drawn as before parallel to yy' to determine the points $u'_1 u_2' u_3'$ &c. If the operations described in this and the preceding article be correctly performed, the results will be identical, but the latter is more liable to error than the former.

93. The curve fte, in Fig. 32, is the coursing joint applicable to the case where the right section of the intrados, Fig. 18, is $al'e$, an arc subtending twice 53° 8′. It is evident that it is desirable to adopt the method of Art. (92), and tn is taken equal to an arc subtending an angle of 49° 35′ 18″ at the centre of a circle to radius Oa, Fig. 33: but the method of Art. (91), is better suited to the case, Fig. 29.

94. The equations to the projection of the coursing joint on the horizontal plane are

$$x = -r \sin \theta; \ y = -r \tan \Omega \log_e \tan \left(\frac{\pi}{4} - \frac{\theta}{2}\right).$$

Draw the lines yy', xx' intersecting at right angles in t', and take $t'x = t'x' = aO = Oe$, Fig. 3, and rule lines parallel to yy' through x and x' Apply the *scale of sines* FG so that the zero of the scale may be on $t'y'$ and 90° on $x'W$; then rule lines opposite the divisions 10° 20° 30° &c. parallel to yy'. Also lines $\alpha\alpha'$ or $\beta\beta'$ must be ruled through the divisions marked C_1 or C_2, Fig. 99. Take $n'\rho = gg'$, Fig. 29*, apply the point B_1 of the scale DE, Fig. 98, to the *point* ρ, and cause the zero to fall on xx', and then through the points 10° 20° 30° &c. draw lines parallel to xx', and thus determine the points $o_1 o_2 o_3$ &c. Or $\nu\sigma$ may be made equal to GG', Fig. 29*, and then the point B_2 in the scale, Fig. 98, must be made to coincide with σ when the zero of the scale falls on xx'.

95. The projection of the same curve on a vertical plane parallel to the abutments will be easily obtained by the

method shewn in Fig. 31, where IH is the scale of cosines, Fig. 100, $n\rho = n\rho' = gg'$, Fig. 29*, and $\nu\sigma = GG'$.

96. The most convenient method of finding the projection of the coursing joint on any vertical plane intersecting the horizontal plane xy in WV is as follows. Having obtained the projections of corresponding points, as $o_7o_6o_5$ &c., $r_7r_6r_5$ &c., Figs. 30 and 31, let fall perpendiculars on VW from $o_7o_6o_5$ &c., Fig. 30. Then n_7r_7, n_6r_6, n_5r_5 &c., Fig. 31, will give the heights of the several points n_7s_7, n_6s_6, n_5s_5 &c. above WV, and thus the points $s_7s_6s_5$ &c. in the required curve will be found.

97. The extradosal coursing joint A_1tA_2 may either be found as directed in Art. (85), or it may be drawn independently in the same way as a_1ta_2 was determined in Art. (91.) In this case ty' must be equal the arc $A'L'$, Fig. 3, and tn must be taken equal an arc of 74° 35′ 8″ to radius R.

98. In Fig. 32, ac is the same development of the intrados as in Fig. 18, fe is the coursing joint. In Fig. 32*, $ft'e$ is the projection of the coursing joint on a horizontal plane, and r'_3le, Fig. 33, is the projection of the same curve on a plane parallel to the abutments.

99. Having found the curves a_1ta_2, and A_1tA_2, pieces of cardboard must be shaped so as to agree accurately with them. In Fig. 34, bc is the development of the intrados, and ....$a_{17}u_{17}$, $a_{16}u_{16}$ &c. are all curves identical in form with a_1ta_2, Fig. 29, and similiarly situated with respect to the line lm. So also are the coursing joints.... $\nu_{17}b_{17}$, $\nu_{16}b_{16}$ &c., in Fig. 40, identical in form with the curve A_1tA_2, Fig. 29, and similarly situated with respect to LM. If corresponding coursing joints be drawn on the intrados and extrados, Figs. 34 and 40, these curves will intersect lm, LM, produced if necessary in some points q, Q, such that $qm = QM$.

100. The plan eca, Fig. 35, is the same as that given in Fig. 4, and amc, Fig. 36, is the projection of the face of the arch over aOc on a vertical plane, parallel to the axis of the cylinder, or parallel to the abutments of the bridge. Suppose $a_1a_2a_3$ &c., Fig. 34, to be the points in which the coursing

joints meet the heading spiral *amc;* then through these points draw $a_1a'_1a_1\gamma_1$, $a_2a'_2a_2\gamma_2$ &c. parallel to *cc*, intersecting *ca*, Fig. 35, in $a'_1a'_2$ &c., and the ellipse, Fig. 36, in $\mathrm{a}_1\mathrm{a}_2$ &c. It is evident that if we suppose the development *bc* to be turned about the edge *dc* and wrapped round the cylinder, Fig. 35, that *amc*, Fig. 34, will be situated in the vertical plane through *cOa*, Fig. 35.

101. The projection of *mu* will be *uO*. Also $a'_1a'_2a'_3$ &c. will then be the projections of $a_1a_2a_3$ &c. on the horizontal plane, and $\mathrm{a}_1\mathrm{a}_2\mathrm{a}_3$ &c. those in the vertical plane parallel to *dc*. Hence if in Fig. 38 we measure $Oa'_3 = Oa'_3$ in Fig. 35, erect a perpendicular $a_3a'_3$ and make it equal to $\mathrm{a}_3\gamma_3$, in Fig. 36, the point a_3 will be the point in the elevation, Fig. 53, corresponding to a_3 on the development, Fig. 34, and so on for the other points. Figs. 40, 41, 42 and 43, belong to the extrados, and are of precisely the same kind as Figs. 34, 35, 36 and 38.

102. Another method is as follows. With the semi-axes major and minor of the ellipse *alc*, Fig. 49, describe arcs of circles *dc*, *ma*, Fig. 37. Through $a_1a_2a_3$ &c. draw lines as in Fig. 39, parallel to *lm*, and apply the scale of equal parts *EF* in the manner indicated, and after reading off the divisions on the scale where these lines meet the edge of it, lay down corresponding angles in Fig. 37, *Om* being the line from which the angles are measured. Let the reading corresponding to a_6 in Fig. 39, be that which expresses the number of degrees in dOs_6, Fig. 37. Let Os_6 meet the circles *dc*, *ma* in s_6 and r_6. Draw s_6a_6 parallel to *dO*, and r_6a_6 at right angles to *dO* intersecting s_6a_6 in a_6; then a_6 is the point in the elevation corresponding to a_6 on the development, Fig. 34. See also Figs. 56 and 57.

103. The points $a_1a_2a_3$ &c. in the elevation will be most accurately determined as follows. In Fig. 38 describe the circles as in Fig. 37, and also draw the elliptic quadrant *mc*. Lay down the angles dOs_9, dOs_8 &c. Now we know that the points sought must lie in the ellipse *mc*. Hence it will suffice to draw lines parallel to *dO* through the points $s_9s_8s_7$

&c. to intersect the ellipse in $a_9a_8a_7$ &c. Or to draw lines through $r_9r_8r_7$ &c. parallel to Oc intersecting the ellipse mc in the required points $a_9a_8a_7$ &c. The latter gives a more correct result in the neighbourhood of ac, and the former in the neighbourhood of md.

Precisely the same processes may be used in finding the several points $b_1b_2b_3$ &c. in the elevation, Fig. 43, corresponding to $b_1b_2b_3$ &c., Fig. 40, $b_5n_5 = M\rho_5$, $b_6n_6 = M\rho_6$ &c. The joints b_1a_1, b_2a_2, b_3a_3 &c., on the face of the bridge, Fig. 53, do not converge to one point, but they are very nearly normals to the ellipse alc at the points where they meet it.

The preceding methods of finding points in the elevation corresponding to points in the developments are manifestly applicable to all cases of the construction of oblique bridges.

104. Having given methods of obtaining the coursing joints and their projections on various planes, we now proceed to consider how proper forms may be given to the arch-stones to be used in the construction of the equilibrated oblique bridge. The straight-edged winding strips of Figs. 78, 79, and 80, can no longer be used. In this case it is advisable to work the cylindrical surface first, and the proper form of $\alpha\beta\delta\gamma$, Fig. 86, can be obtained from the development Fig. 34. The arch-square gfe must be applied as directed in Art. (63), Fig. 86. Thus the beds $\alpha\beta$, $c\delta$ will be obtained. Take the common mason's square DEF, Fig. 96, and apply the stock DE along one of the generating lines of the cylinder, then draw the normal a_6n_6. The lines αa, βb, γc, δd, may be found in this manner in Fig. 86. The ends of the stone must be so formed that if $\beta r = \delta q$, the straight line rq joining r and q may be on the surface of the stone.

105. In order to determine the forms of the quoins: The complete developments, of which LPC, Lpc are the halves, Fig. 56, must be described as directed in Art. (69), but instead of the straight lines ap, aP, the coursing joints ta_1, tA_1, of Fig. 29 must be drawn corresponding to every

joint in *ae* and *AC*, Fig. 40. Through the point p, where the intradosal coursing joint meets the intrados, draw *pn* parallel to *LE* to meet the extradosal coursing joint *aP*, or *aP* produced, in n. Then suppose that in Fig. 86, *ad* is a quoin properly formed on the equilibrated system on the sides *ae*, γd, βa, and $\gamma\beta$, and that the two points a_{22} and a_{21} have been formed: draw the normals $a_{22}n_{22}$, $a_{21}n_{21}$, and supposing that p in *Lc* is the point a_{22}, P will be the point b_{22}, and *Pn* will be the length of $n_{22}b_{22}$ measured on the extradosal coursing joint *cd*. Thus three points, a_{21}, a_{22} and b_{22}, become known in the plane face of the quoin.

106. There can be no doubt as to the superiority of the equilibrated over the spiral courses for an oblique bridge, when the arch-stones are supposed to be formed with *equal accuracy*. If we examine Fig. 50, we see that the courses are inclined at an angle ϕ to the horizon immediately above the impost *dc*, where that part of the arch is nearly vertical: but on the other hand, in an equilibrated bridge adapted to the same circumstances in Fig. 54, the courses tend to become horizontal, as they no doubt ought to do. Again, on this equilibrated system there is great difficulty in the arrangement of the courses in the neighbourhood of the acute angles a and d of the abutments, where they become very much reduced in thickness. For this reason it seems to be desirable to avoid arches having semi-circles for their direct section, both on the spiral and equilibrated systems; but the latter admits of a nearer approximation to the semi-cylinder than the former.

107. Mr. Chapman constructed oblique bridges in Ireland in 1787, and his system appears to have been distinct from either of the preceding. The coursing joints differed slightly from straight lines; but if *ah* had been drawn at right angles to the tangent to the heading spiral at m, Fig. 8, and if the coursing joints had been drawn parallel to *ah*, a near approximation would have been made to Mr. Chapman's system. The consequence would have been that ϕ, the angle of the intrados, would have been increased from

about 23° to 40°, and the evil of having the courses of the vertical part of the arch, Fig. 50, inclined to the horizon would have been very much aggravated. When however the right section is a segment, as in Fig. 18, the difference is by no means so important. Mr. Chapman has described his methods in the article *Oblique Arches* which he contributed to Rees's *Cyclopædia*.

The instrument, Fig. 69, consists of a light frame with holes *bbb*.... in which a number of cylindrical pins *aaa*.... of *equal* lengths may be inserted and clamped in any position. If now (*bb*) be put in the place to be occupied by an arch-stone, and if the pins (*a*) be clamped so that one end of each may be in contact with the beds of the stones already set, then the opposite ends of the pins (*a*) will be points in the surface proper for the bed of the arch-stone intended to occupy the position of (*bb*).

Fig. 55, is a plan of the Figs. 49 and 50. The curves a_6a_6, a_7a_7 &c. were drawn by a card-board pattern formed to agree accurately with *ath*, Fig. 17; and $a_6a_7a_8$ &c. are the projections of $a_6a_7a_8$ &c., Fig. 50, on the horizontal plane. For the plan of the intradosal coursing joints, Fig. 51 and 52, the curve *ath*, Fig. 24, and for Figs. 53 and 54, the curve $\sigma' t' \sigma$, Fig. 30.

PART III.

ON THE CONSTRUCTION OF OBLIQUE ENDS OF DIRECT ARCHES.

108. It sometimes happens that although the two faces of a bridge may be vertical planes, yet these are not parallel, and consequently they would intersect in a vertical straight line if they were produced, as in the plans, Figs. 72 and 76. Fig. 75 is a plan of a bridge with oblique ends, the arch over $a'c'd'b'$ is built exactly as a direct arch, but the courses, after passing $c'a'$, $d'b'$, begin to curve as indicated in the plan. The mode in which I have obtained these lines is as follows. Produce $c'a'$, $b'd'$, which are supposed to be at right angles to the axis of the soffit, so as to meet ca, bd produced in X, X' respectively, and let $a'X = d'X' = d$. Suppose any vertical plane whatever to pass through X and intersect the soffit over $acc'a'$, and let the intradosal coursing joints be determined so that they may all cut at right angles this intersection of the plane and soffit.

109. If $y'x$, $y'y$ be the axes of X and Y, the equations to the development $a_3t_3\beta_3$,

$$Y = r\theta,\ X = r \cot \Omega \text{ vers } \theta + a_0a_3,\ \text{Art. (39)},$$

$$= r \cot \Omega \text{ vers } \theta + d \cot \Omega \ldots\ldots\ldots (13).$$

Let $y = f(x)$ be the equation to a coursing joint on the development of the soffit which cuts a_3t_3 at right angles at the point xy. Then we must have at their point of intersection,

$$y = Y, \quad x = X, \quad \text{and } \frac{dy}{dx} = -\frac{dX}{dY} = -\frac{r \cot \Omega \sin \frac{Y}{r}}{r}$$

$$= -\frac{x \sin \frac{y}{r}}{r \operatorname{vers} \frac{y}{r} + d},$$

$$\therefore x \frac{dx}{dy} = -\frac{r \operatorname{vers} \frac{y}{r} + d}{\sin \frac{y}{r}}:$$

and integrating, we have

$$\frac{1}{2}x^2 + C = -r \int_y \frac{\sin \frac{y}{2r}}{\cos \frac{y}{2r}} - \frac{d}{2} \int_y \frac{\sec^2 \frac{y}{2r}}{\tan \frac{y}{2r}}$$

$$= 2r^2 \log_e \cos \frac{y}{2r} - rd \log_e \tan \frac{y}{2r} \ldots (14).$$

Suppose that $y = r\theta$ as before, and then we get the equations to the coursing joint, under the form

$$y = r\theta, \quad \frac{1}{2}x^2 + C = 2r^2 \log_e \cos \frac{\theta}{2} - rd \log_e \tan \frac{\theta}{2} \ldots\ldots (15).$$

110. Now there may be an infinite number of curves (15), drawn to intersect (13), for all values of Ω, at right angles. Suppose however that we confine our attention to the coursing joint gg_1g_2, and let $y'g = y = r\alpha$, then manifestly $x = 0$, and we get

$$0 + C = 2r^2 \log_e \cos \frac{\alpha}{2} - rd \log_e \tan \frac{\alpha}{2},$$

which determines C; and the required equation becomes x^2

$$= \left(4r^2 \log_e \cos \frac{\theta}{2} - 2rd \log_e \tan \frac{\theta}{2}\right) - \left(4r^2 \log_e \cos \frac{\alpha}{2} - 2rd \log_e \tan \frac{\alpha}{2}\right)$$

$$= \frac{4r^2}{\log_{10} e} \left\{\left(L_{10} \cos \frac{\theta}{2} - \frac{d}{2r} L_{10} \tan \frac{\theta}{2}\right) - \left(L_{10} \cos \frac{\alpha}{2} - \frac{d}{2r} L_{10} \tan \frac{\alpha}{2}\right)\right\}$$

$$\left.\begin{aligned}&\text{or } x^2 = \frac{4r^2}{.4343}\left\{f\left(\frac{\theta}{2}\right) - f\left(\frac{\alpha}{2}\right)\right\}\\&\text{and } y = r\theta\end{aligned}\right\}\ldots\ldots\ldots\ldots\ldots\ldots(16),$$

where θ and α may have any values between 0 and π; and $\therefore \frac{\theta}{2}$ and $\frac{\alpha}{2}$ have 0 and $\frac{\pi}{2}$ for their limits. In (16) α and $\therefore f\left(\frac{\alpha}{2}\right)$ is constant for *any given coursing joint* $gg_1g_2\ldots$ and is such that $r\alpha$ = the distance $y'g$ at which the coursing joint intersects the right section of the soffit made by the plane passing through X.

111. The values of x in equation (16), cannot be determined by the methods of Fig. 29, and it is somewhat troublesome to perform the calculation. It is however worthy of remark, that from the nature of the equations (16), it is evident that if once a figure similar to the development a_0acc', Fig. 70, were carefully laid down and engraved, that this would be applicable to all bridges whatever where $\frac{d}{r}$ is a constant ratio. Such a plate might have very numerous coursing joints, printed in a neutral tint with the developments $\alpha_1\beta_1$, $\alpha_2\beta_2$, $\alpha_3\beta_3\ldots$, corresponding to the sections of the soffit made by vertical planes passing through $\alpha_1\beta_1$, $\alpha_2\beta_2$ &c. Nothing more would then be required than to trace over the lines in black that were required for any particular bridge. $tc = ty' = r\,\frac{\pi}{2}$ = an arc subtending an angle of 90° at the centre of a circle of radius r. If the right section of the proposed arch were segmental, take $ta' = t\gamma' = \frac{1}{2}$ the arc subtended by the angle aOe, Fig. 18, at the centre of the right section to radius Xy', Fig. 70, and through α', γ' draw lines $\alpha'\alpha$, $\gamma'\gamma$ parallel to lm meeting the development corresponding to the given value of the obliquity in α, γ. Then $\alpha'\alpha\gamma\gamma'$ is the required development, and the coursing joints already drawn there are quite as applicable to this case as to any other, for which $\frac{r}{d}$ remains constant. $r'r''$ is equal

the radius r used in Fig. 70, and all the development $\gamma'\gamma a a'$ may be enlarged or diminished in any of the ordinary ways in which plans are enlarged or reduced.

112. If the development Fig. 57, be formed into its proper cylindrical surface to radius r, so that lm may be in a vertical plane passing through the axis of the cylinder, the equations to a coursing joint will be

$$y = r \operatorname{vers} \theta,\ z = r \sin \theta, \text{ and } x = 3.035r \sqrt{\left\{f\left(\frac{\theta}{2}\right) - f\left(\frac{\alpha}{2}\right)\right\}} \ldots (17).$$

Where the axis of x is the intersection of the cylindrical surface and the horizontal plane passing through its axis, the axis of y is at right angles to it in the same horizontal plane, and the axis of z is vertical.

The equations to the projection of the coursing joint in (17), on a horizontal plane, are

$$y = r \operatorname{vers} \theta, \quad x = 3.035r \sqrt{\left\{f\left(\frac{\theta}{2}\right) - f\left(\frac{\alpha}{2}\right)\right\}}, \text{ Fig. 73} \ldots (18),$$

and on a vertical plane parallel to the abutment or parallel to the plane yz,

$$z = r \sin \theta, \quad x = 3.035 \times r \sqrt{\left\{f\left(\frac{\theta}{2}\right) - f\left(\frac{\alpha}{2}\right)\right\}} \ldots (19).$$

113. The equations to the extradosal spiral corresponding to (17) will evidently be

$$Y = R \operatorname{vers} \theta,\ Z = R \sin \theta,\ X = 3.035r \sqrt{\left\{f\left(\frac{\theta}{2}\right) - f\left(\frac{\alpha}{2}\right)\right\}} \ldots (20).$$

The equations to the projection of this curve on the horizontal plane XY are

$$Y = R \operatorname{vers} \theta, \quad X = 3.035r \sqrt{\left\{f\left(\frac{\theta}{2}\right) - f\left(\frac{\alpha}{2}\right)\right\}} \ldots (21);$$

and on a plane parallel to the abutments,

$$Z = R \sin \theta, \quad X = 3.035r \sqrt{\left\{f\left(\frac{\theta}{2}\right) - f\left(\frac{\alpha}{2}\right)\right\}} \ldots (22).$$

The equations to the above extradosal coursing joint, $A'A$, $A'C'$, Fig. 71, being the axes of X and Y, and R the radius of the extrados, are

$$Y = R\theta, \quad X = 3.035r\sqrt{\left\{f\left(\frac{\theta}{2}\right) - f\left(\frac{a}{2}\right)\right\}}, \text{ Fig. 71..(23).}$$

114. If we suppose a to have a constant value, all the preceding equations relate to the same coursing joint. Thus if $\left(\frac{a}{2}\right) = 85°$, $ra = y'g$, Fig. 70, and $Ra = A'G$, Fig. 71, and equations (17) and (20) are the equations to the corresponding intradosal and extradosal coursing joints $gg_1g_2g_5$, and $GG_1G_2G_5$. If in addition we give θ the same values in equations (17) and (20), we shall have $x = X$, and we shall obtain the coordinates of contemporaneous positions of p_0 and P_0, Fig. 9* Art. (83), on the developments, Figs. 70 and 71.

115. Let ac, AC be the intradosal and extradosal heading spirals of an oblique bridge, and g_1g_5, G_1G_5 corresponding coursing joints. Through g_5 draw g_5n parallel to cX. Also in Fig. 71, draw G'_5N parallel to $C'X$, so that $tn = TN$; then $G_5G'_5$ will be the distance $n_{22}b_{22}$ to be measured as in Fig. 85, in the formation of the quoin.

116. If we draw straight lines $\tau\sigma\rho$ parallel to cX, and then take points τ such that $\rho\tau : \rho\sigma :: R : r$, Fig. 70, the points τ will be points in the extradosal coursing joint, corresponding to σ on the intradosal coursing joint. It is convenient in this case to make a_0x the axis of x rather than lm, as we have done in former cases. The values of $\frac{a}{2}$ marked along $y'c$, $A'C'$, Figs. 70, 71, and 73, are useful in the original drawing, but for practical applications the graduations BB' must be used. For instance, in the example, Fig. 18, the angle $aOl' = 53°\ 8'$, and through these divisions in the columns, BB' in Figs. 70, 71, and 73, straight lines $\gamma'\gamma$, $a'a$ must be drawn parallel to lm. Also in the above example the roads are supposed to be inclined at an angle $\Omega = 50°$,

and therefore $a'\alpha\gamma\gamma'$ is the development of the intrados of the arch to radius $r'r''$. Fig. 74 exhibits the same drawing enlarged in the ratio $r'r''$, Fig. 70: $r_1'r_1''$, Fig. 74.*

117. In equation (16) we substitute

$$f\left(\frac{\theta}{2}\right) \text{ for } L_{10}\cos\frac{\theta}{2} - \frac{d}{2r}L_{10}\tan\frac{\theta}{2}.$$

In the figures of Plate XI., d has been supposed equal to r, or or that $Xa' = a't'$, Fig. 73. It is manifest that for a given obliquity the smaller d or Xa' is made, the less will be the amount of oblique arching; but if d be too much diminished, the arch-stones in the neighbourhood of aa_0 may be unable to support the pressure thrown upon them. The developments $a_0a_4\beta_4c$, Fig. 70, and $a'acc'$, Fig. 74, are adapted to the same plan, Fig. 4, but the right section of the soffit is in the former case the semi-circle $al'e$, Fig. 3, and in the latter an arc $al'e$, Fig. 18, subtending at O an angle equal twice 53° 8′. By increasing the radius the bearing surface at the obtuse abutment is increased for $a'a$, Fig. 74, is greater than a_0a_4, Fig. 7.

118. In performing the numerical calculations of

$$f\left(\frac{\theta}{2}\right) = L_{10}\cos\frac{\theta}{2} - \frac{1}{2}L_{10}\tan\frac{\theta}{2},$$

$\frac{\theta}{2}$ was made successively equal to 1°, 2°, 3° &c. 89°, and the values of $f(1°)$, $f(2°)$, $f(3°)$. . . . $f(89°)$ were easily found by the help of logarithmic tables. At the same time the values of $f\left(\frac{a}{2}\right)$ were found. Suppose now that $\frac{a}{2} = 85°$, then it is found that $f\left(\frac{a}{2}\right) = f(85°) = 3.4112$, and it is constant for every point in $gg_1g_2g_5$, the equations to which are $y = r\theta$, $x = 3.035\ r\sqrt{\left\{f\left(\frac{\theta}{2}\right) - 3.4112\right\}}$. And by giving to $\frac{\theta}{2}$ any value 83°, $y = 2r \times$ circular measure of $83° = 2r\theta$, and $f\left(\frac{\theta}{2}\right) = f(83°) = 3.6305$ has been already calculated. Hence

$x = 3.035\, r \sqrt{}(3.6305 - 3.4112) = 3.035\, r \sqrt{}(.2193)$. By giving to $\frac{\theta}{2}$ other values, other points in the coursing joint gg_s may be found.

119. It is evident that the foregoing methods of determining coursing joints is applicable to bridges constructed on the plans, Figs. 72 and 76.

120. The development of the soffit of the arch required in the plan, Fig. 72, would be of the form $\beta_1 a_1 a_4 \beta_4$, Fig. 70. Suppose that a general drawing, Fig. 70, on a large scale has been prepared, and let it be required to apply it to a particular case where the obliquity is Ω. From X draw Xa_4 making an angle $90° - \Omega$ with Xa_0, and then in the ordinary way obtain the development $a_4 t_4 \beta_4$, if it has not been laid down already on the drawing. If the right section be a semicircle, $B'a_4\beta_4 B$ will be the required development. If otherwise, $a'a$, $\gamma\gamma'$ must be drawn through the divisions on BB' corresponding to the number of degrees &c. in half the angle subtended by the right section aOe, Fig. 18. Then $\gamma'\gamma a a'$ will be the development drawn to such a scale that $y'X$ is the length of the radius of the right section of the arch. The drawing $a'a\gamma\gamma'$ and its coursing joints may be enlarged, or an appropriate scale may be applied to it. It is manifest, from the mode in which the coursing joints have been drawn, that it is of little importance to the stability of the bridge, whether the arch be terminated by $a_2 t_2 \beta_2$ or $a_4 t_4 \beta_4$, because $\beta_2 a_2 a_4 \beta_4$ would stand by itself.

A few properties of the ellipse which are likely to be useful to the draughtsman may be here noticed. In Fig. 27, let AA', BB' be the axes major and minor of an ellipse, O being its centre. With radius OA or OA' and centre B describe an arc cutting AA' in s, s': these two points s, s' are called the foci of the ellipse. With centre s and radius Ap describe an arc; and again, with radius $A'p$, and centre s'

describe another arc cutting the one just drawn in P, P': these will be two points in the ellipse of which the axes are AA', BB'. In the same way any number of points may be found in the required ellipse.

In Fig. 28, join $A'B$, and draw BG, $A'I$ at right angles to BA' from the points B, A', meeting respectively AA' in G and BB' produced in I; then OG is the radius of curvature at A, A'; and OI is the radius of curvature at B, B'.

Let pb', qa' be two straight lines intersecting at right angles in O, Fig, 46, and pq a straight line moving so that the point q remains in $a'q$, and p in pb'; then any fixed point r in pq will describe an ellipse of which the semi-axes major and minor are pr, rq. Fig. 45 shews how this property may be applied in order to determine any number of points in the elevation of a bridge. The straight-edge (e) coincides with the axis major aa, and the edge of the set-square (f) with the axis minor, pr is equal in length to the semi-axis minor, and rq to the semi-axis major. If (g) be moved about so as to have its ends p and q in contact with the straight-edge and set-square, r will always give a point in the required ellipse.

In Fig. 44* the two fixed points Q, P in the straight line QPR are supposed to continue in the two lines AA', BB' intersecting at right angles in O. Any point R will describe an ellipse of which the semi-axes are RP, RQ. This is the principle of the elliptic compasses. The method of Fig. 46 is applicable to ellipses of a smaller size than Fig. 44*, but unfortunately the point r will always be over the base of the instrument. In order to avoid this difficulty the arrangement of Figs. 47 and 48 may be adopted when small complete ellipses are required. The bar CD is rigidly fixed to the drawing-board and $\alpha\beta'$, $\gamma\delta'$, are parallelograms, and therefore $P\gamma'$ is always parallel to CD. The point P in $\gamma'P$ is caused to describe the ellipse $ab'a'b$ by being attached to the fixed point P in qp, Fig 48. Consequently any other point P' in $\gamma'\delta'$ will describe another equal and similarly situated ellipse aa', Fig. 47.

In Fig. 44, let *OA*, *OB* be the semi-axes major and minor of an ellipse, and suppose it required to find a number of points in it. With centre *O* and radii *OA*, *OB*, describe circles *AE*, *aB*. Through *O* draw a number of straight lines, of which suppose *Ors* to be one cutting the two circles in *r* and *s*. Through *r* and *s* draw lines respectively parallel and perpendicular to *OA* intersecting in *p*, then *p* will be a point in the required ellipse.

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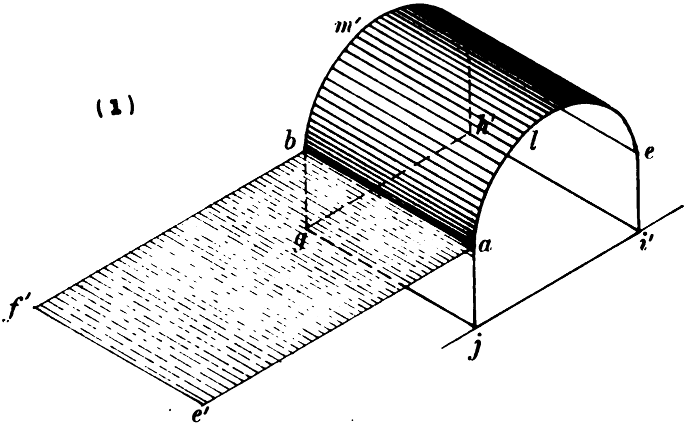
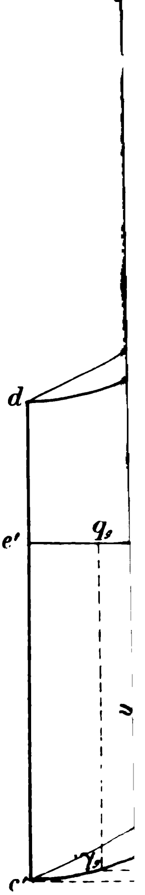
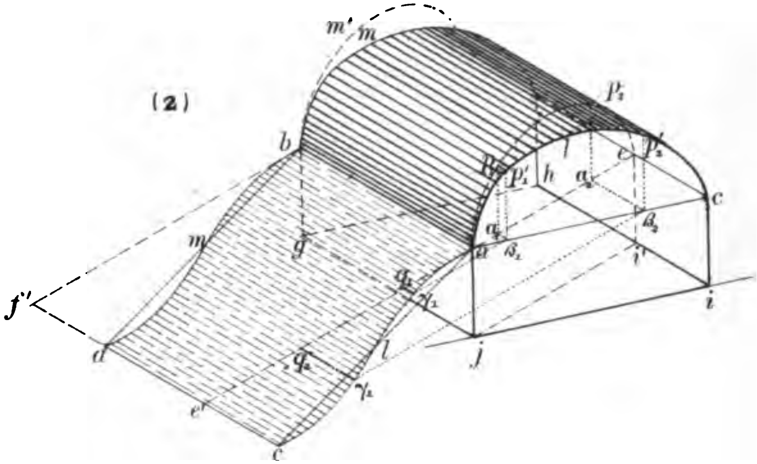
(1)


m'
h'
b
l
e
q
a
i'
f'
j
e'


d
e'
q_9
u
γ_9
c'

(2)


m'
m
P_1
b
l
e
P_2'
B_1
P_1'
h
a_2
c
m
a_1
β_1
β_2
g
i'
a
f'
q_1
γ_1
i
j
d
q_2
l
γ_2
e'
c

(4)

(4")

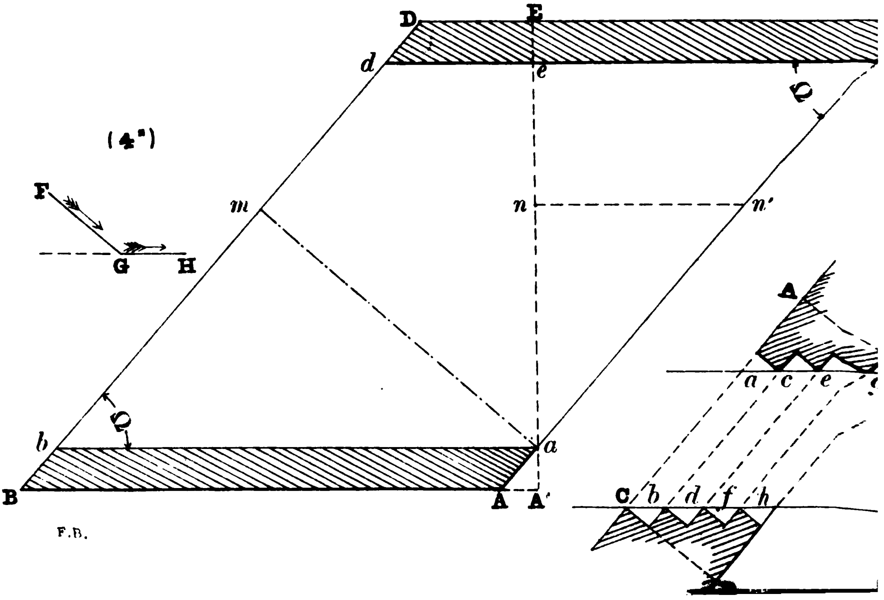
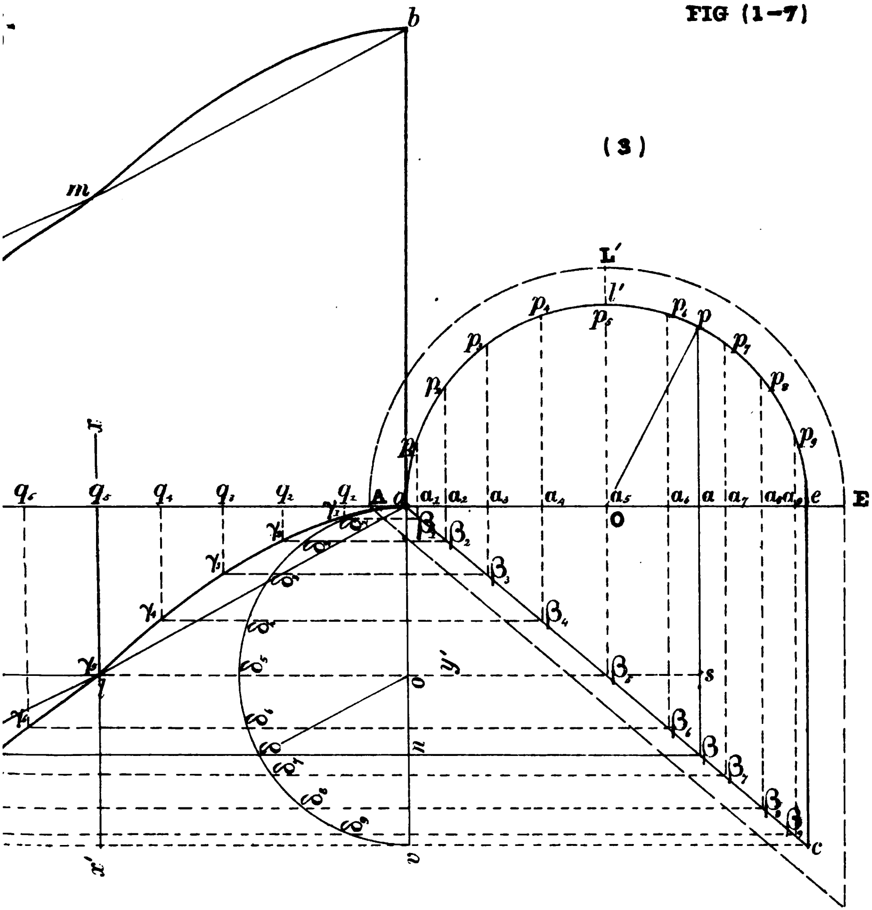
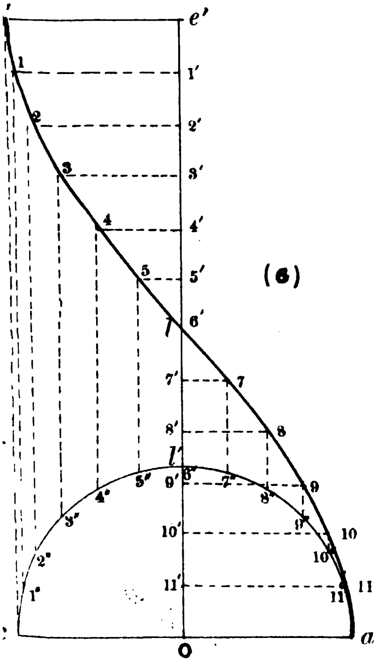

D
E
d
e
Q
F
G
H
m
n
n'
A
a
c
e
Q
b
a
B
A
A'
C
b
d
f
h
F.B.

FIG (1-7)



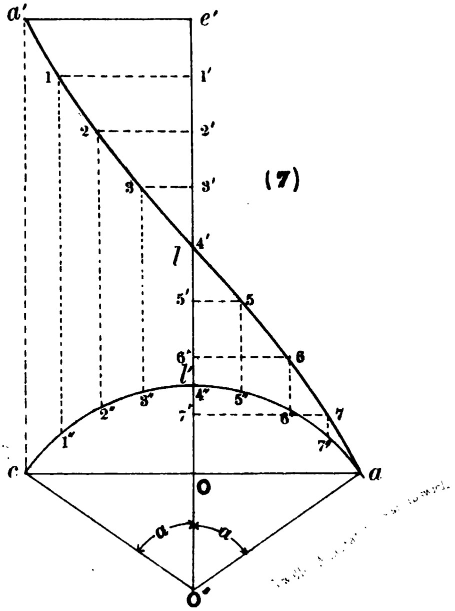
(3)

b
m
L'
l'
p
p₁
p₂
p₃
p₄
p₅
p₆
p₇
p₈
p₉
q₁
q₂
q₃
q₄
q₅
q₆
A
a
a₁
a₂
a₃
a₄
a₅
a₆
a₇
a₈
a₉
e
E
O
β
β₁
β₂
β₃
β₄
β₅
β₆
β₇
β₈
β₉
γ
γ₁
γ₃
γ₄
γ₅
γ₆
δ₁
δ₅
δ₆
δ₇
δ₈
δ₉
y'
s
o
n
v
x
x'
l
c
C



(6)

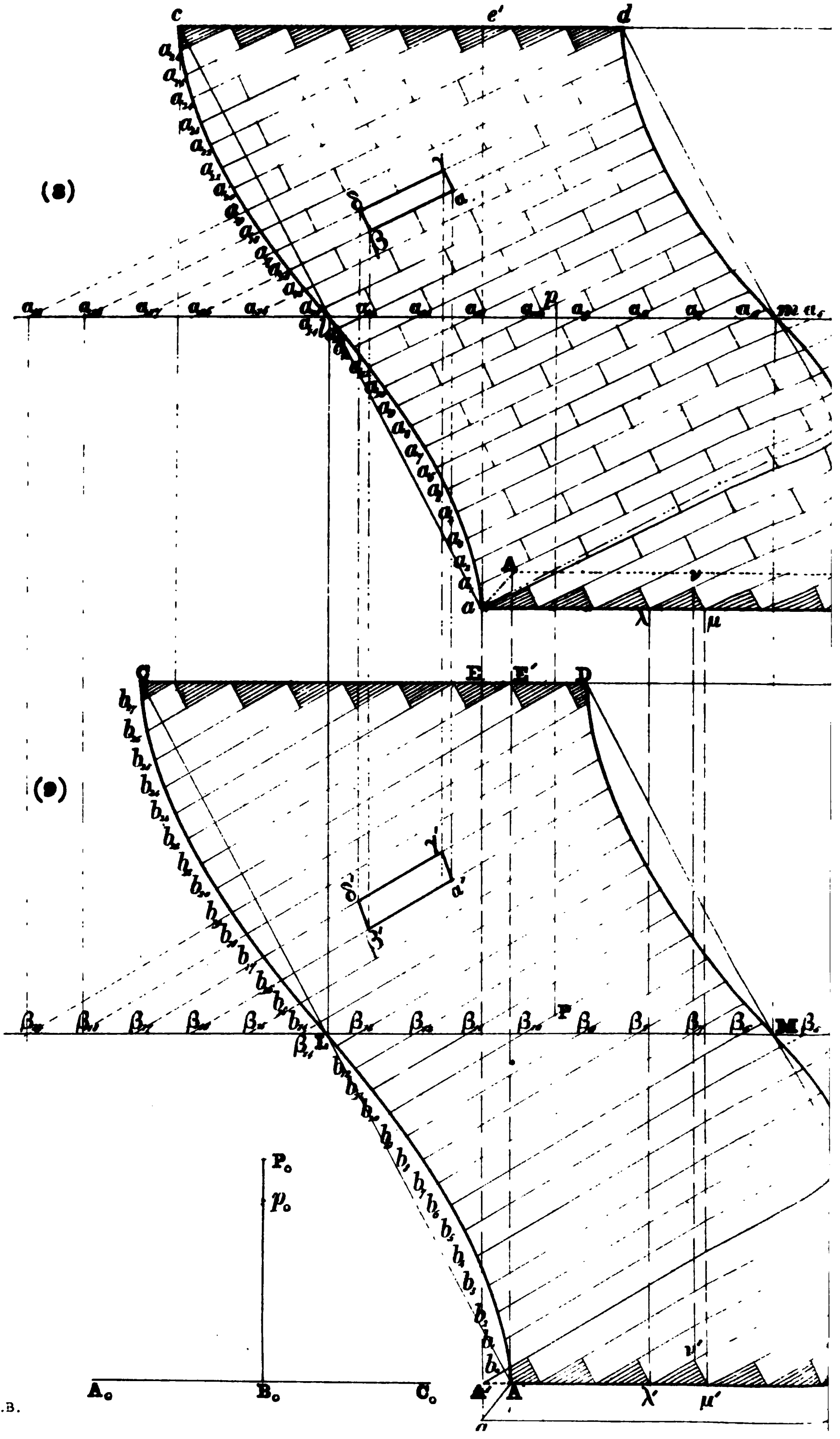
e'
1
2
3
4
5
7
8
9
10
11
1'
2'
3'
4'
5'
6'
7'
8'
9'
10'
11'
l
l'
1"
2"
3"
4"
5"
6"
7"
8"
9"
10"
11"
O
a



(7)

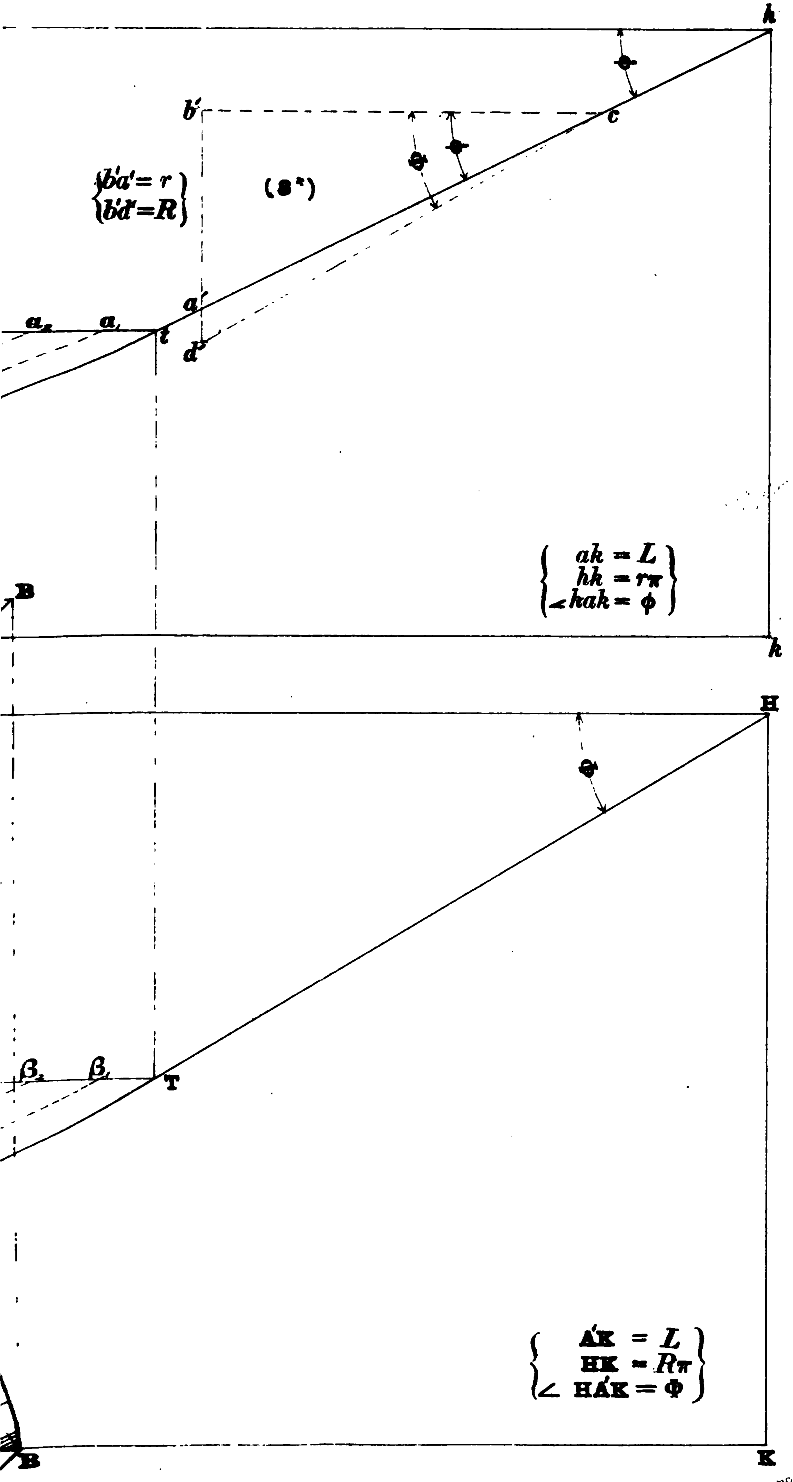
a'
e'
1
2
3
5
6
7
1'
2'
3'
4'
5'
6'
7'
l
l'
1"
2"
3"
4"
5"
6"
7"
c
O
a
α
α
O'

PLATE 2



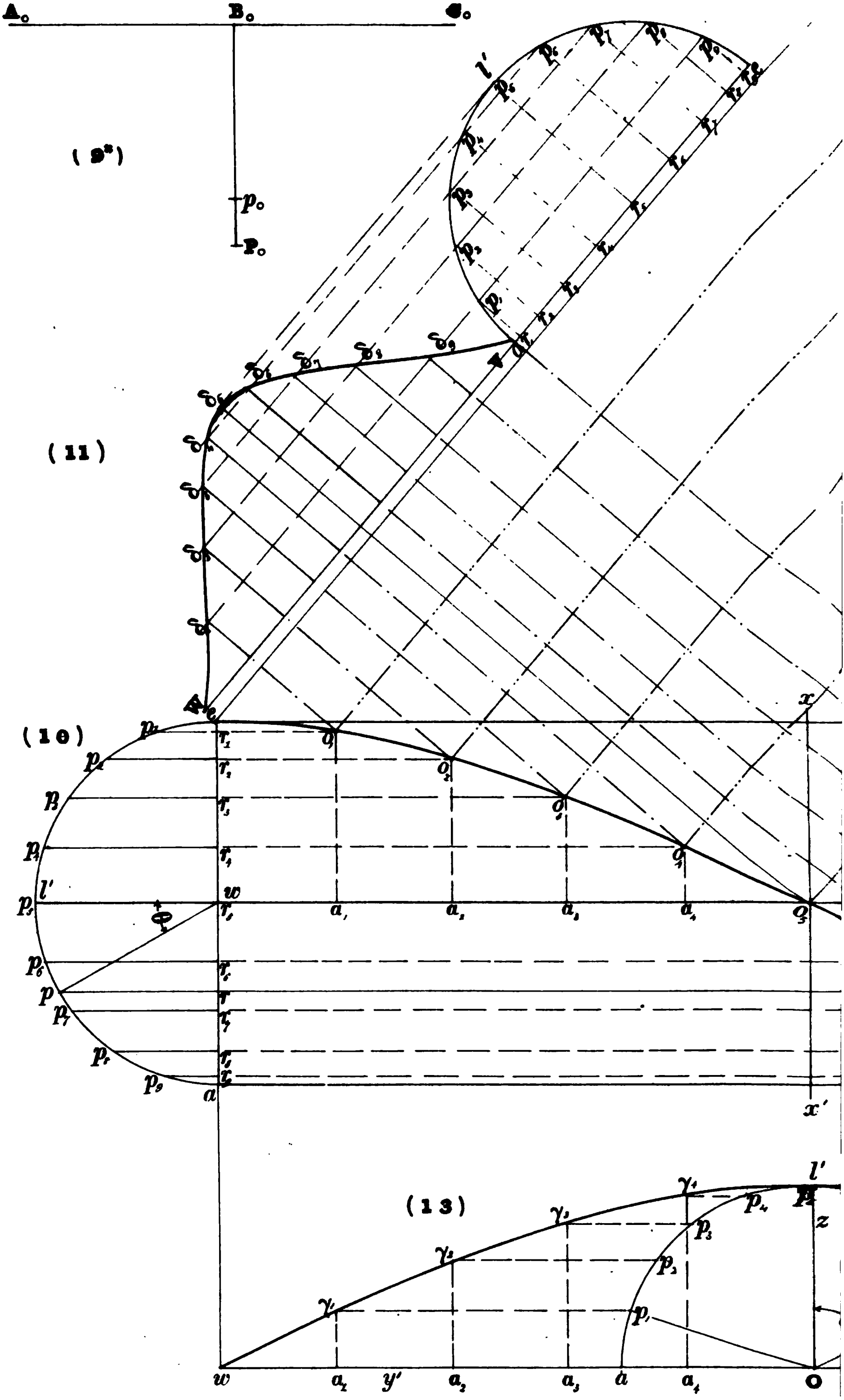
F.B.

FIG (8, 9)



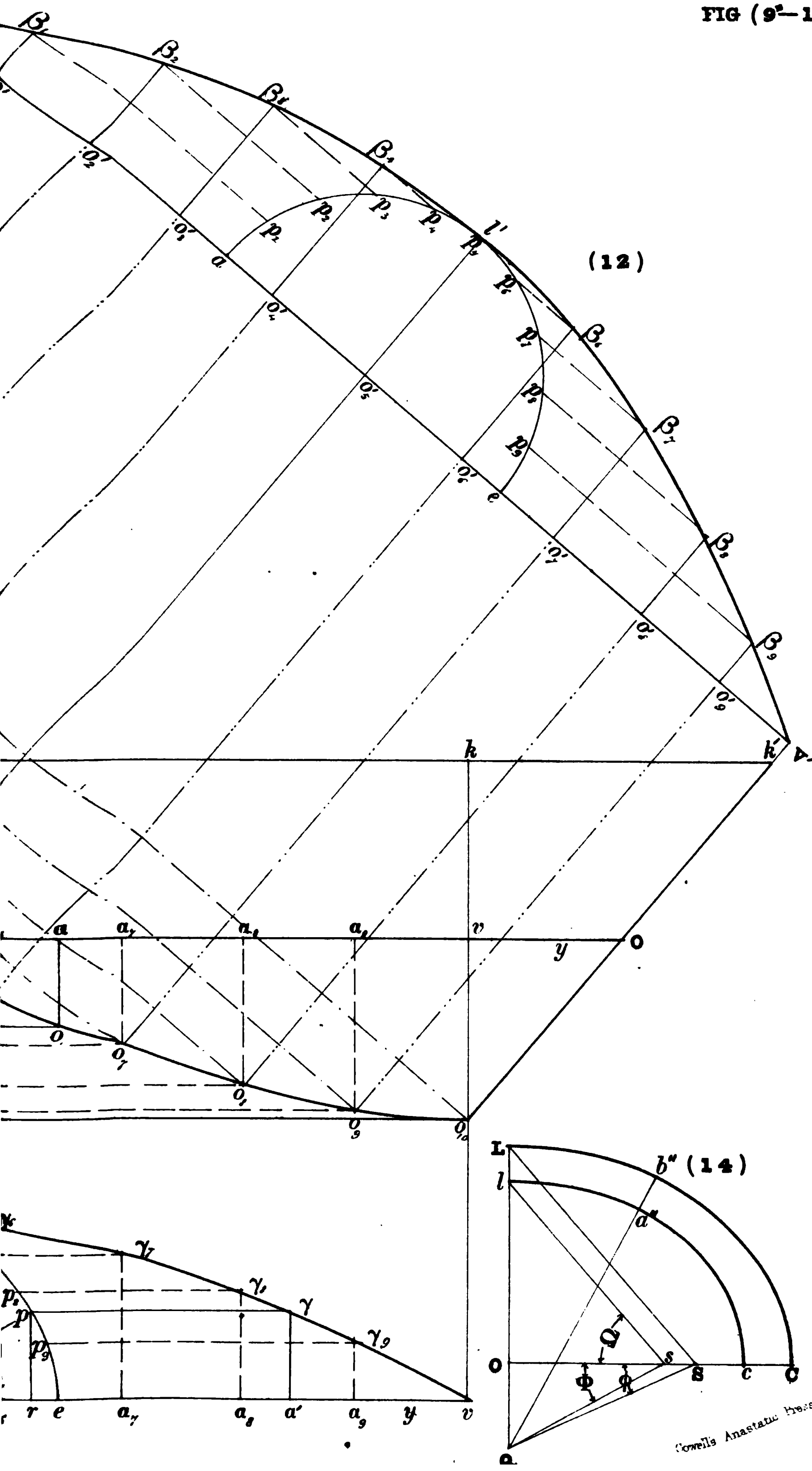
static Press Ipswich

PLATE 3



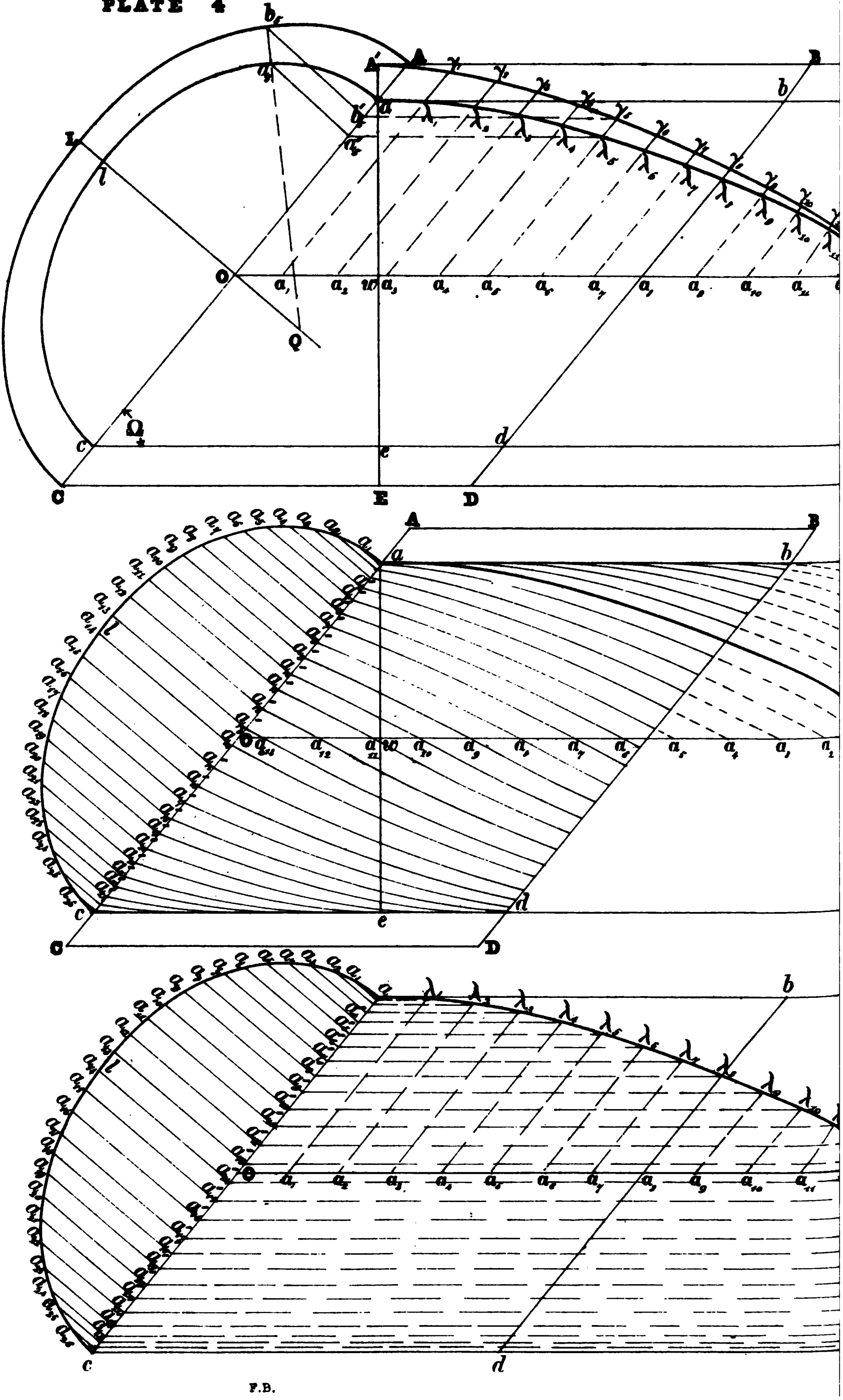
P.B.

FIG (9*—14)



Cowell's Anastatic Press, Ipswich

PLATE 4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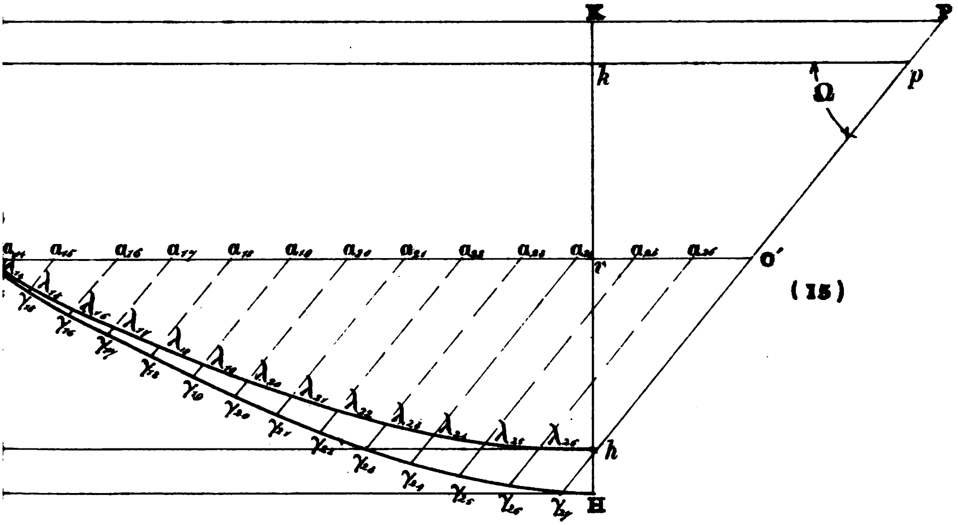
a_5
A'
A
B
γ_1 γ_2 γ_3 γ_4 γ_5 γ_6 γ_7 γ_8 γ_9 γ_{10}
b
a
b'
a'
λ_1 λ_2 λ_3 λ_4 λ_5 λ_6 λ_7 λ_8 λ_9 λ_{10} λ_{11}
L
l
O
a_1 a_2 w a_3 a_4 a_5 a_6 a_7 a_8 a_9 a_{10} a_{11}
Q
Ω
c
e
d
C
E
D
A
B
a
b
l
O
a_{13} a_{12} a_{11} w a_{10} a_9 a_8 a_7 a_6 a_5 a_4 a_3 a_2
c
e
d
C
D
a
b
λ_1 λ_2 λ_3 λ_4 λ_5 λ_6 λ_7 λ_8 λ_9 λ_{10}
l
O
a_1 a_2 a_3 a_4 a_5 a_6 a_7 a_8 a_9 a_{10} a_{11}
c
d

F.B.

FIG (15—17)



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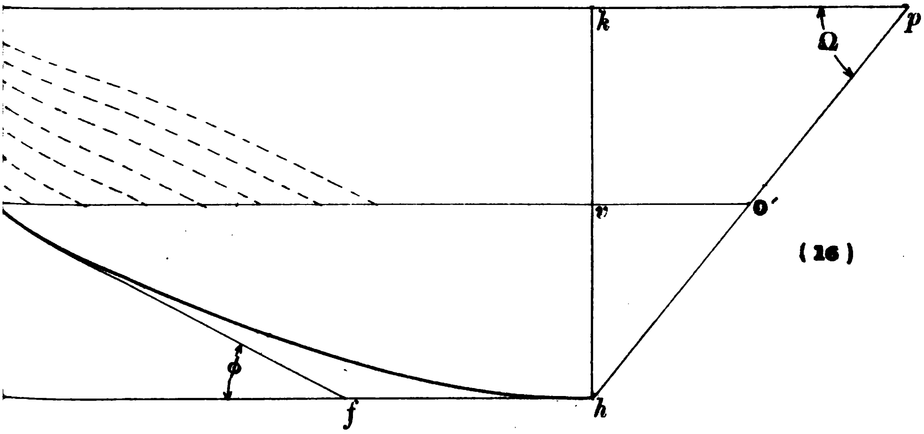

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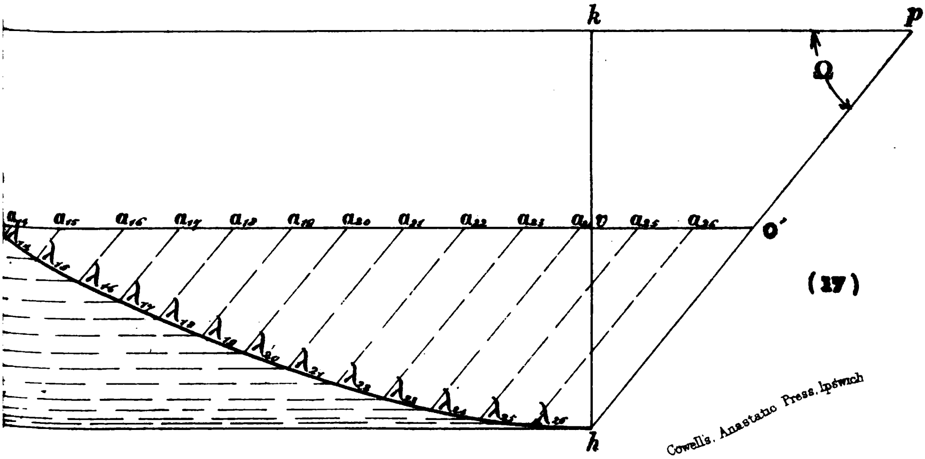
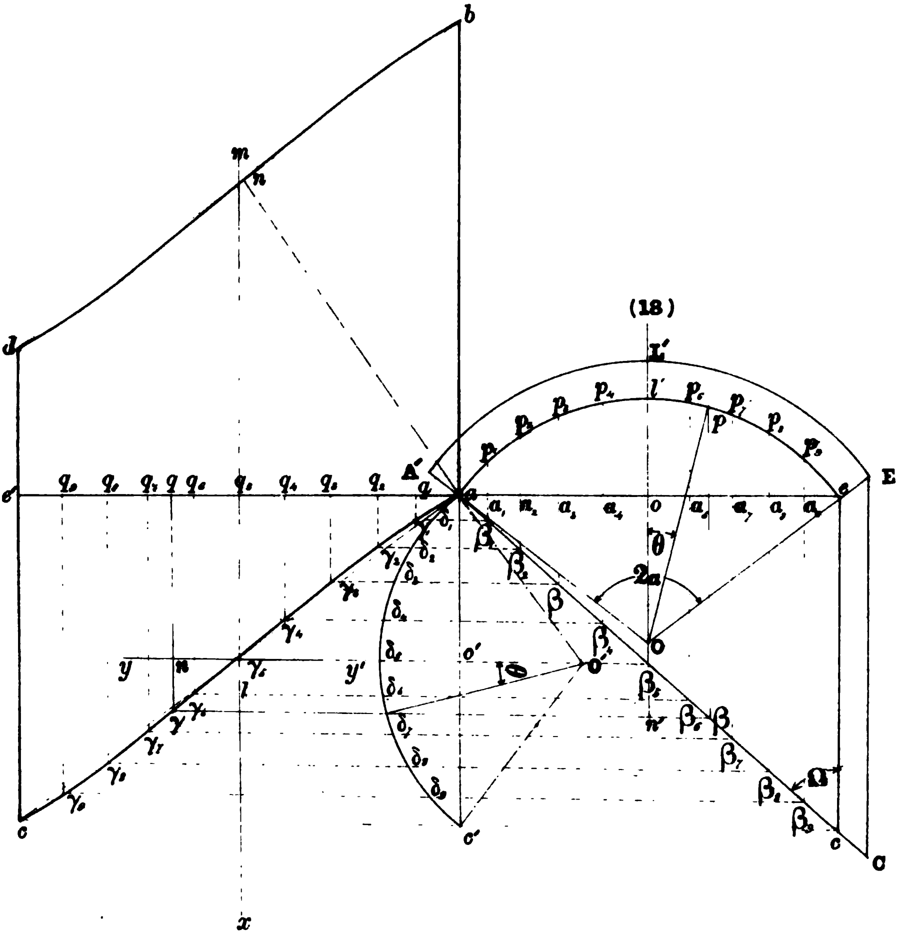


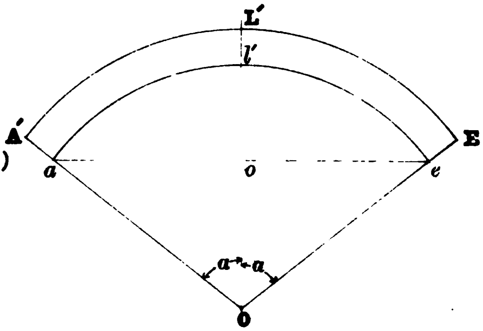
(17)

Cowell's, Anastatic Press, Ipswich

PLATE 5

(18)


b
m
n
d
e
q_9 q_8 q_7 q q_6 q_5 q_4 q_3 q_2 q_1
A′
a
a_1 a_2 a_3 a_4 o a_6 a_7 a_8 a_9
e
E
L′
l′
p_1 p_2 p_3 p_4 p_6 p_7 p_8 p_9 p
θ
2a
O
γ_1 γ_2 γ_3 γ_4 γ_5 γ_6 γ_7 γ_8 γ_9 γ
δ_1 δ_2 δ_3 δ_4 δ_5 δ_6 δ_7 δ_8 δ_9
β_1 β_2 β β_4 β_5 β_6 β β_7 β_8 β_9
o′
O′
θ
y
n
l
y′
n′
Ω
c
c′
c
C
x



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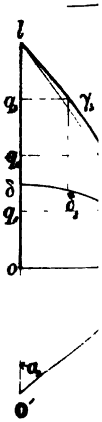

(19)

FIG (18—21)

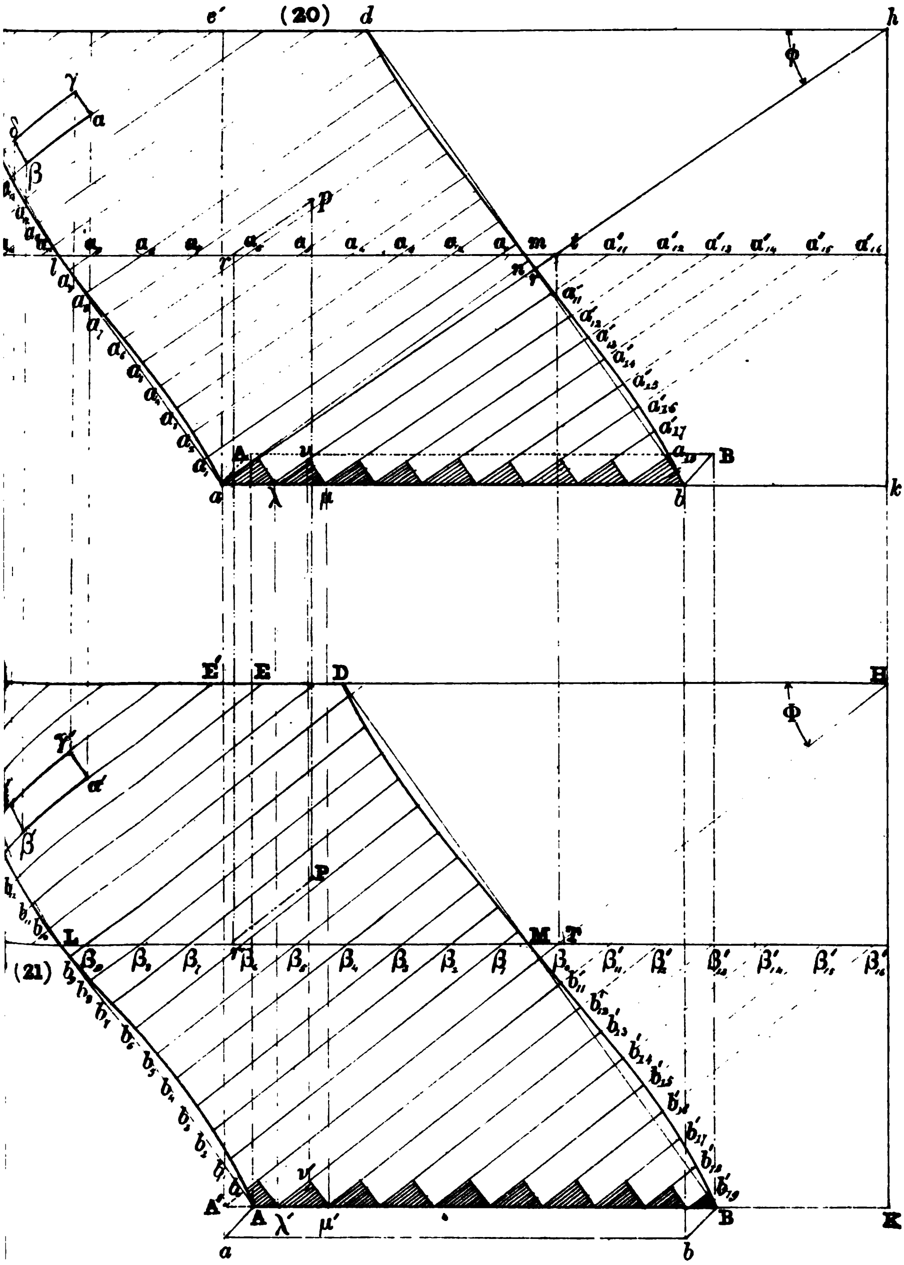

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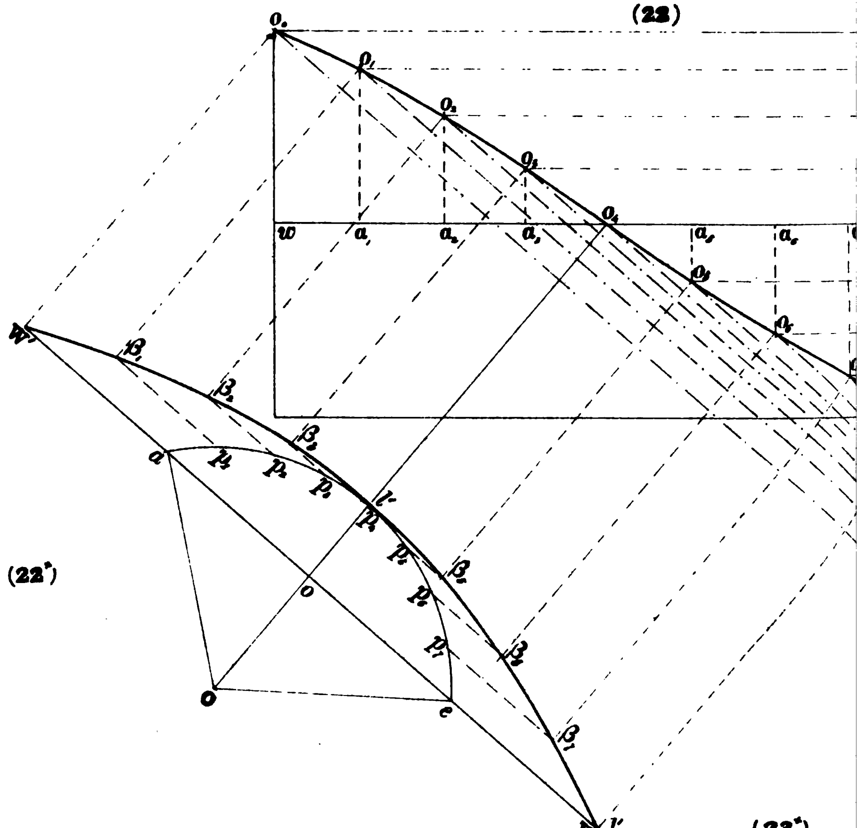
PLATE 6



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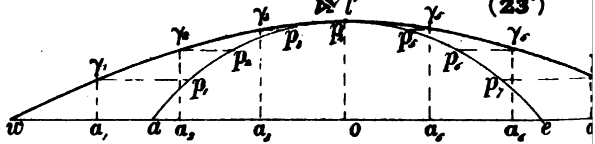

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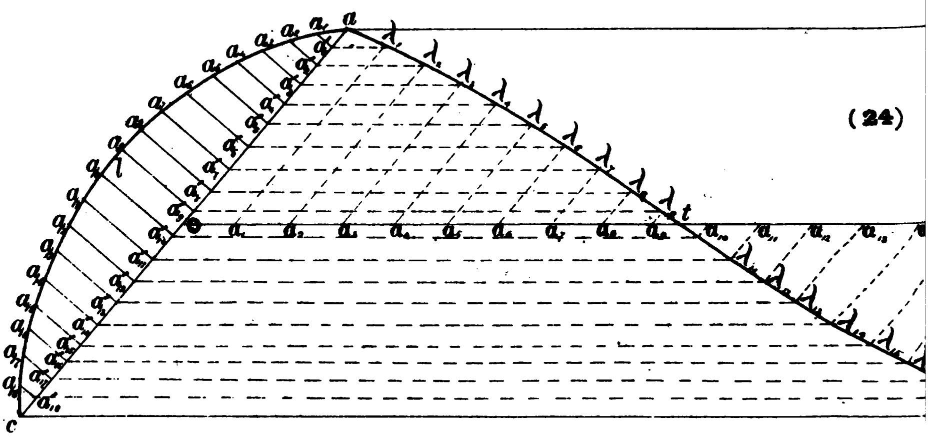
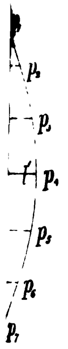


(24)

.B.



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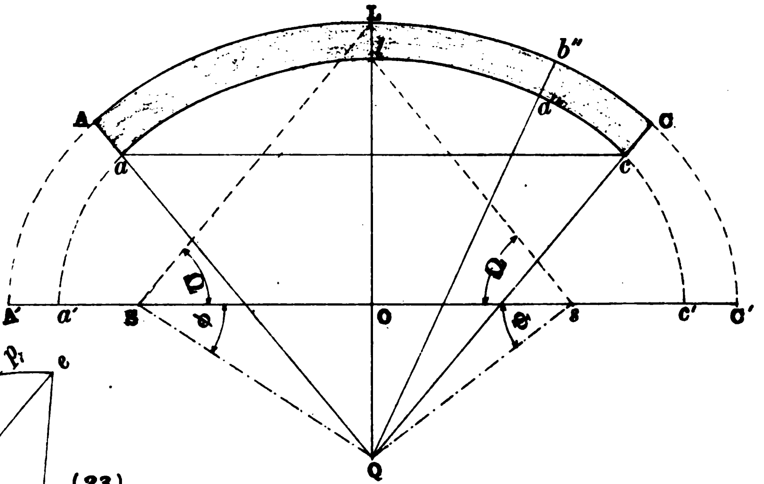

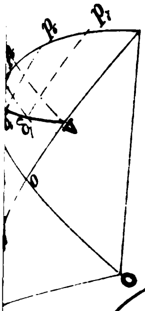

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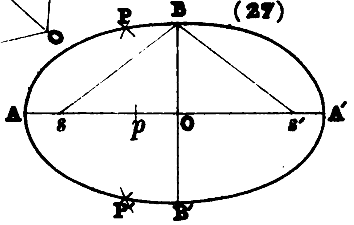

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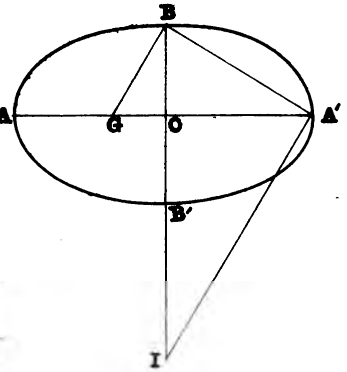

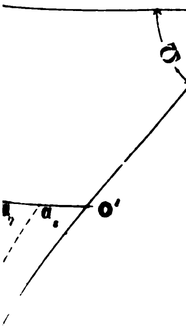

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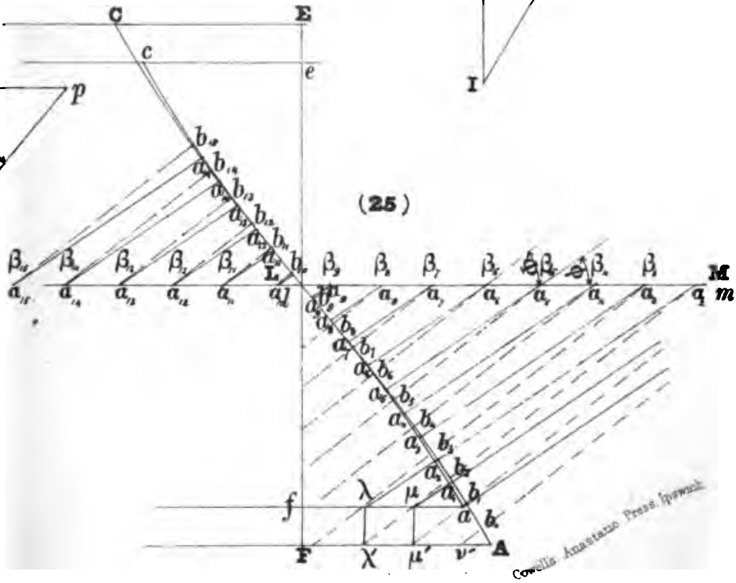

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PLATE 7

E Q U I L I B R A T

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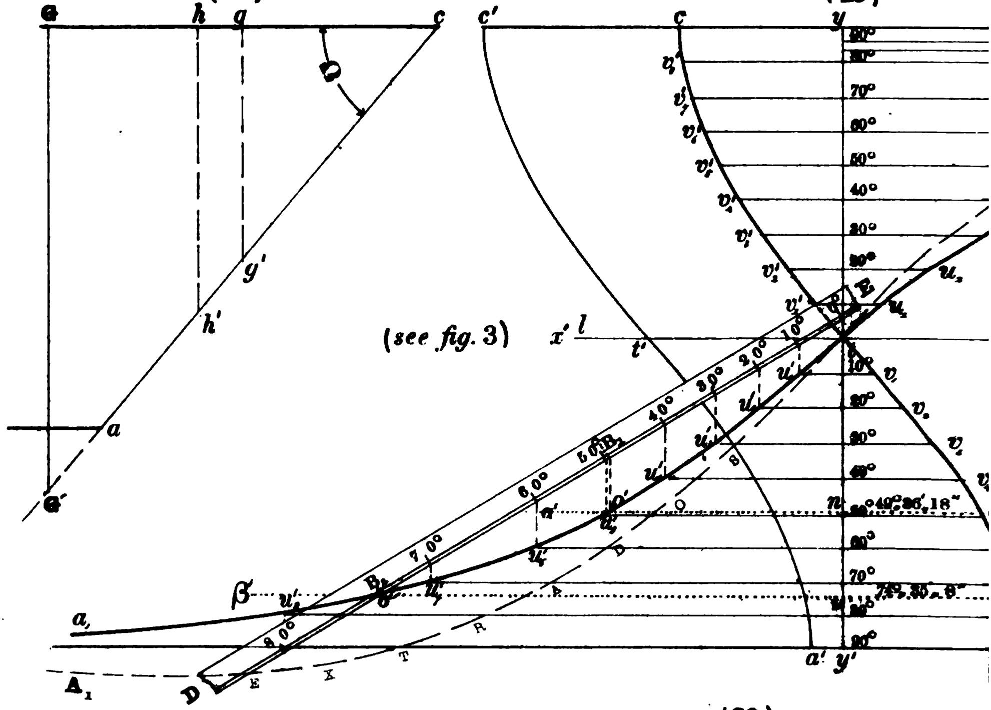
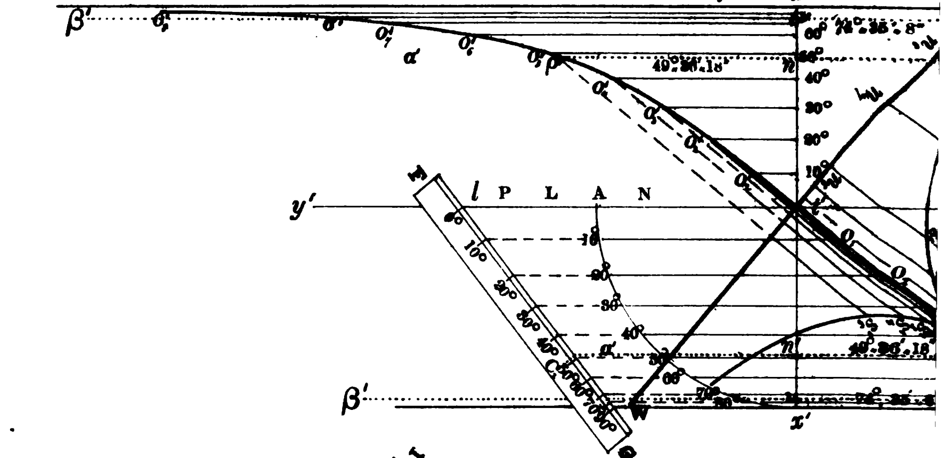


(30)



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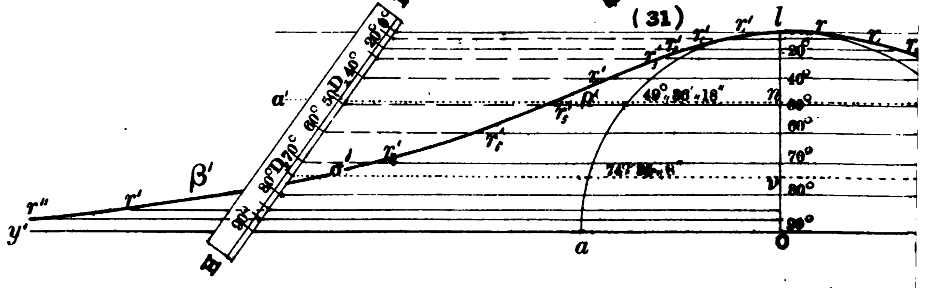
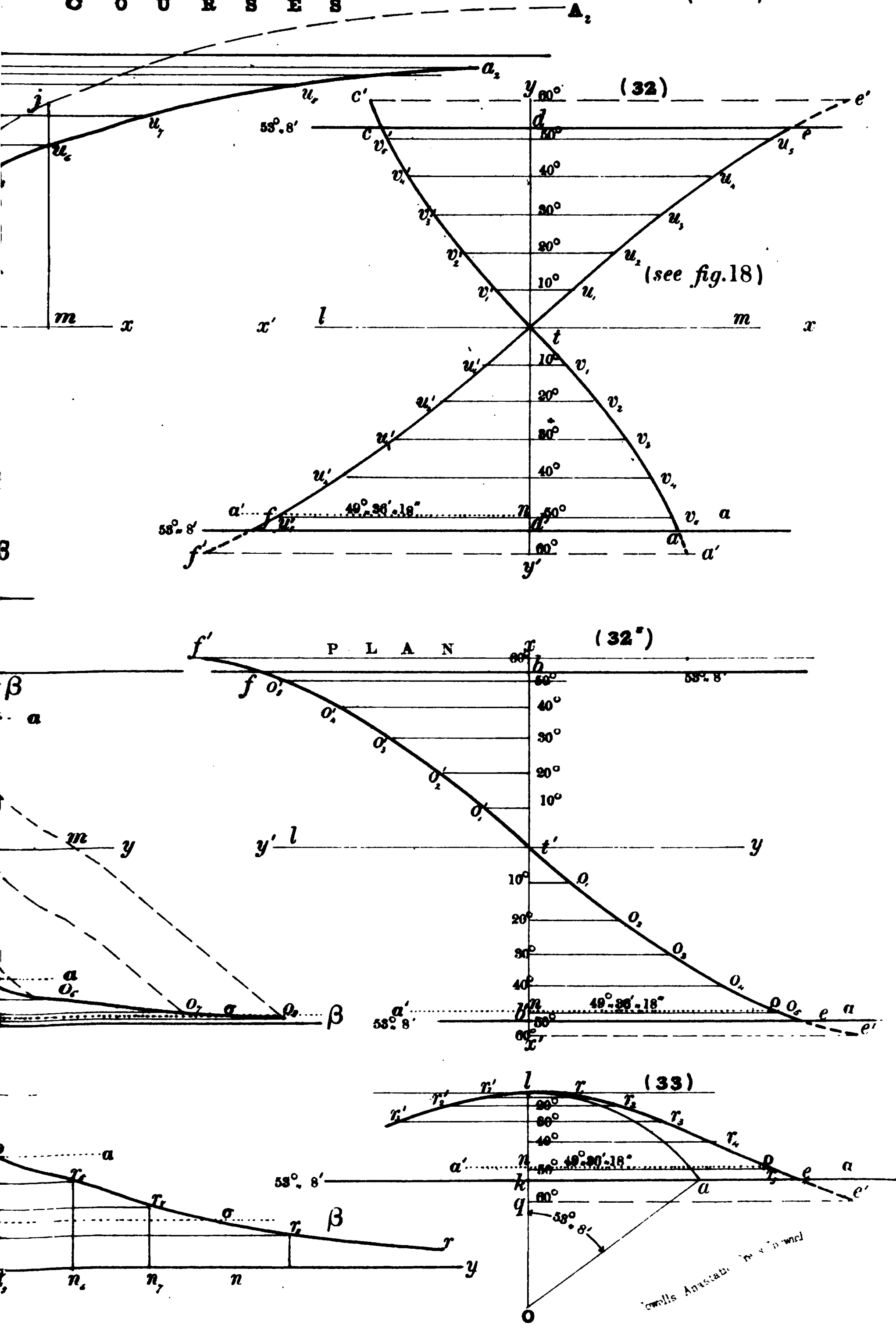


F.B.

C O U R S E S

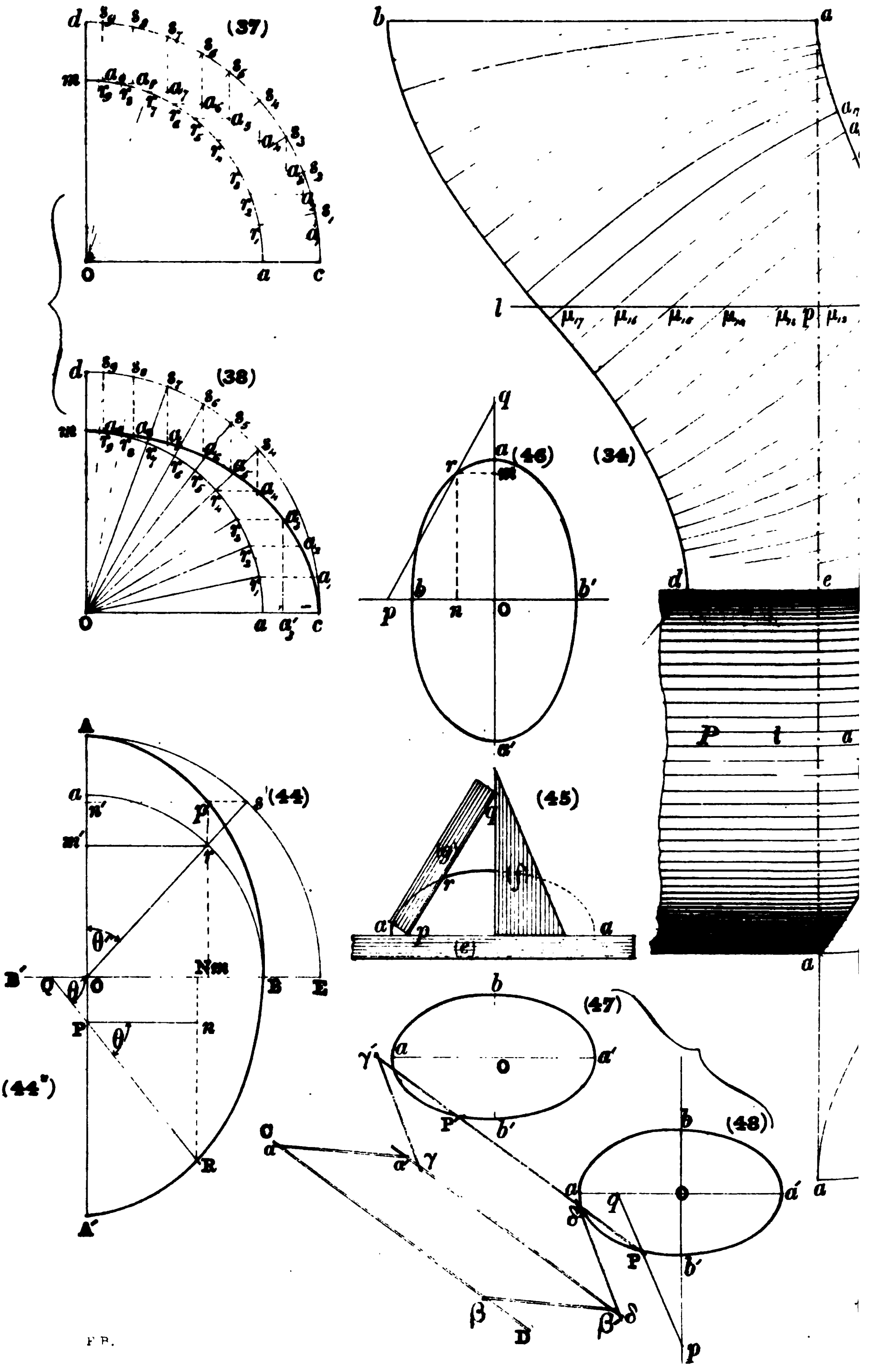
FIG (29-33)



Cowells Anastatic Press Ipswich

PLATE 8

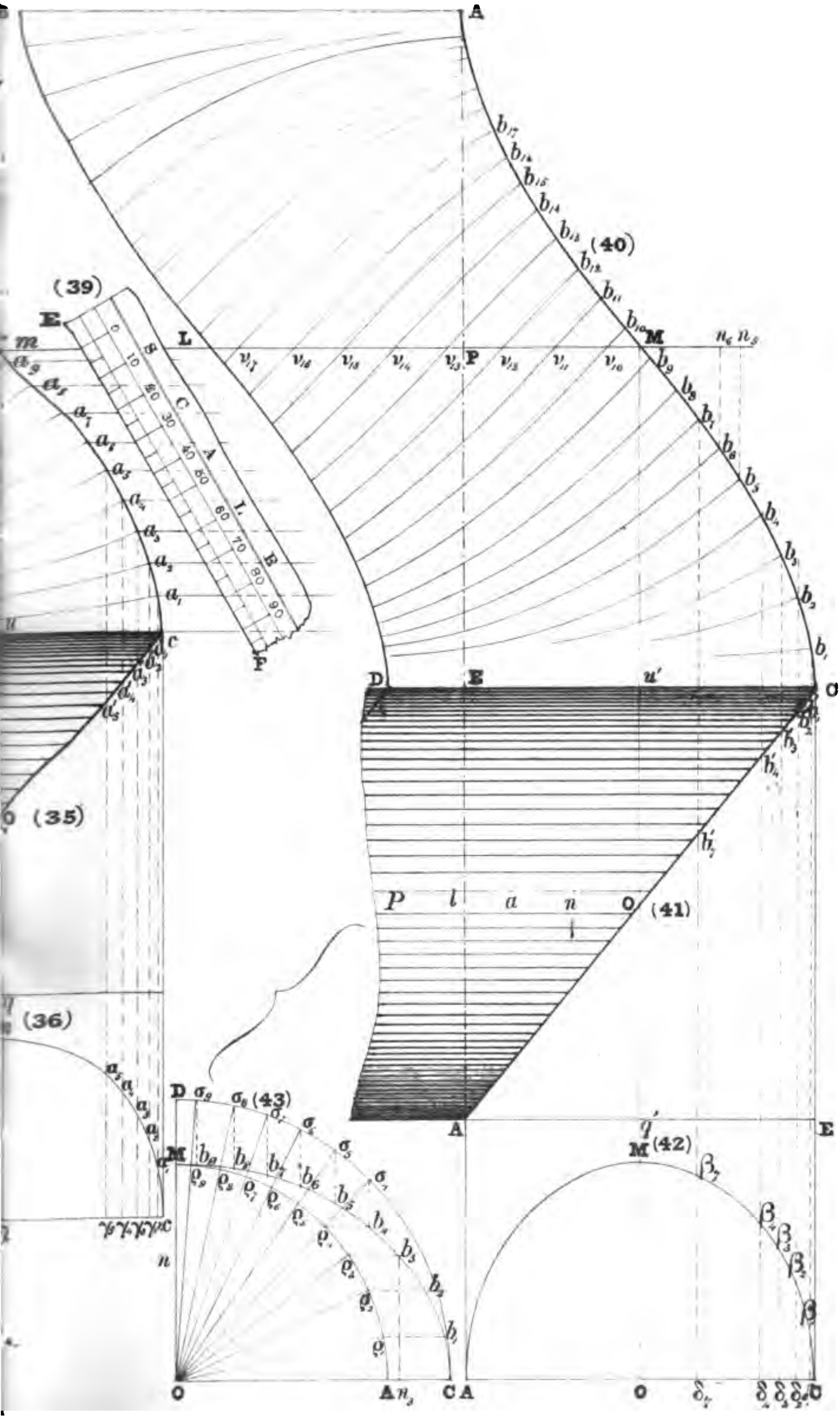
INTRADOS



F R.

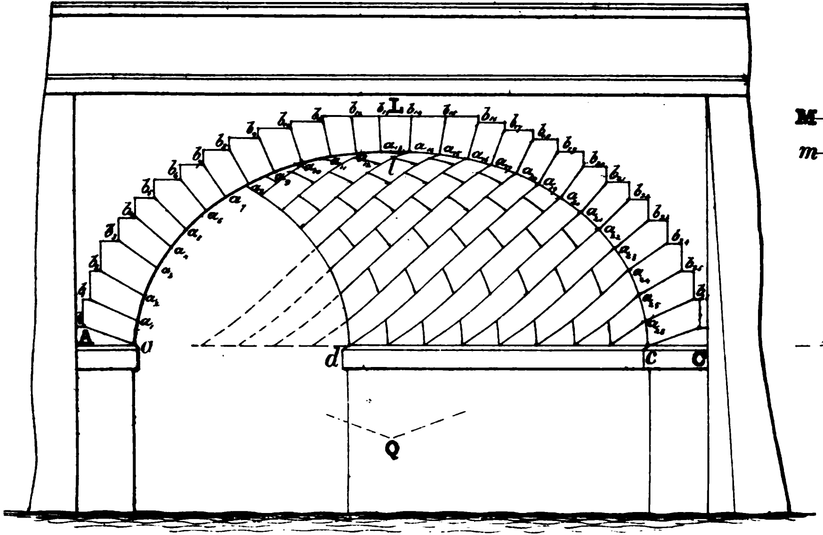
EXTRADOS

FIG (34—48)



SPIRAL COURSES

(49)



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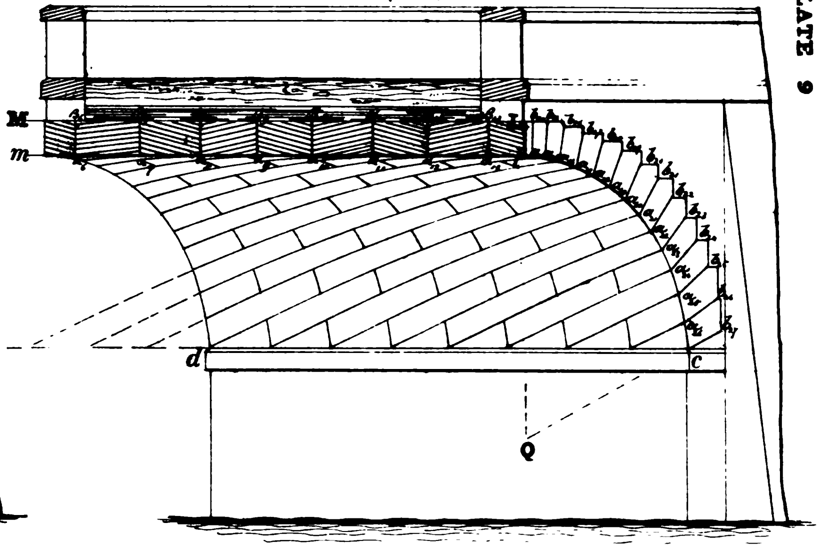
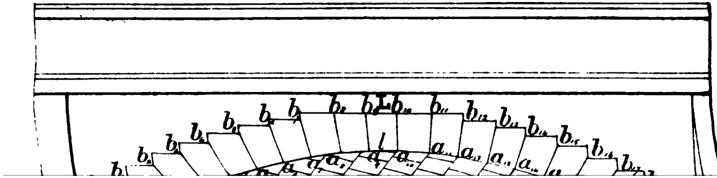


SPIRAL COURSES

(51)



(52)

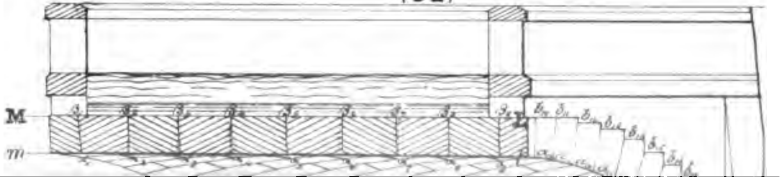
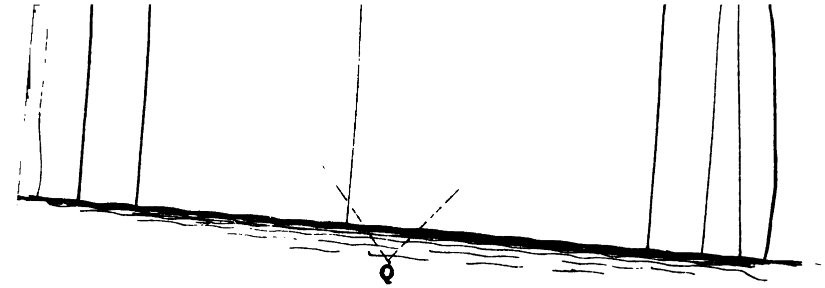
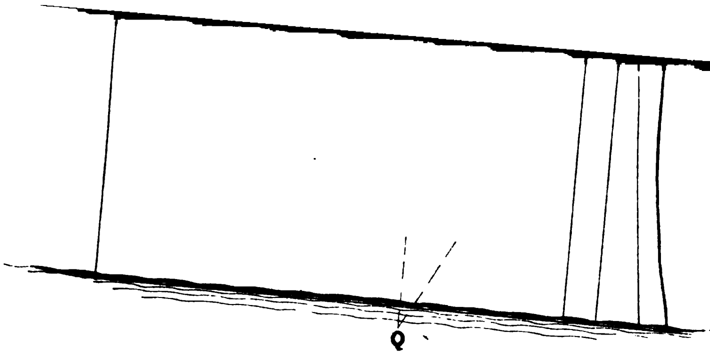


FIG (49–54)

EQUILIBRATED COURSES





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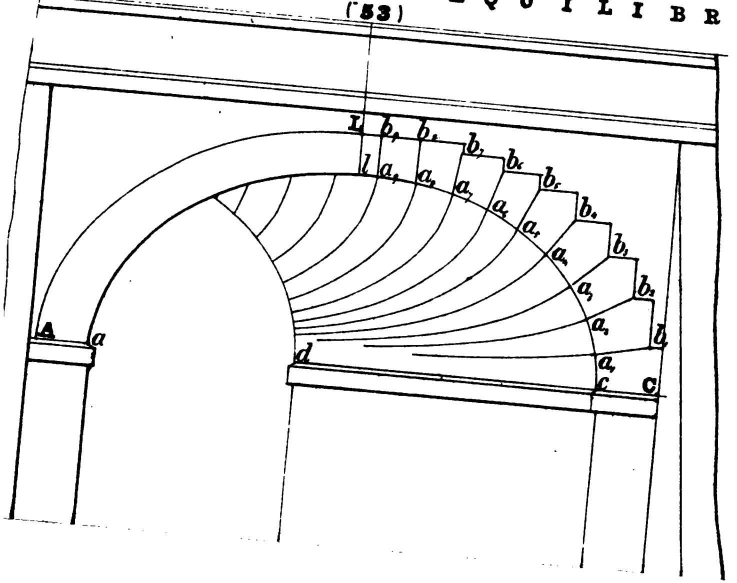

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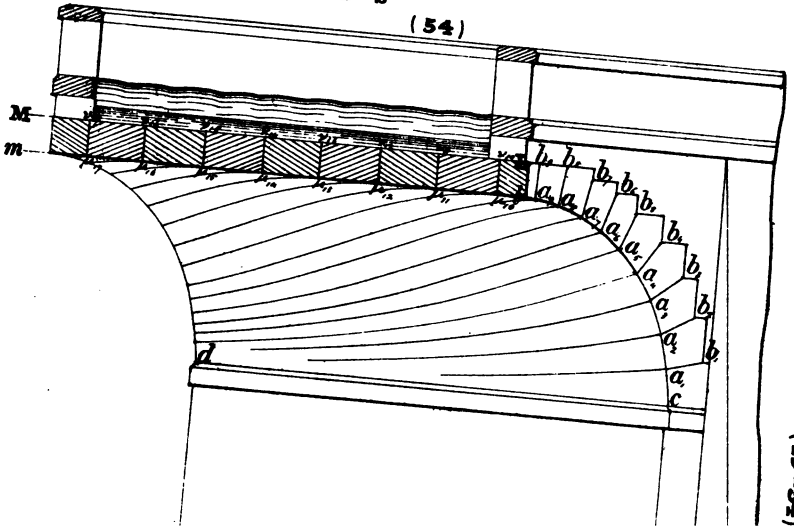
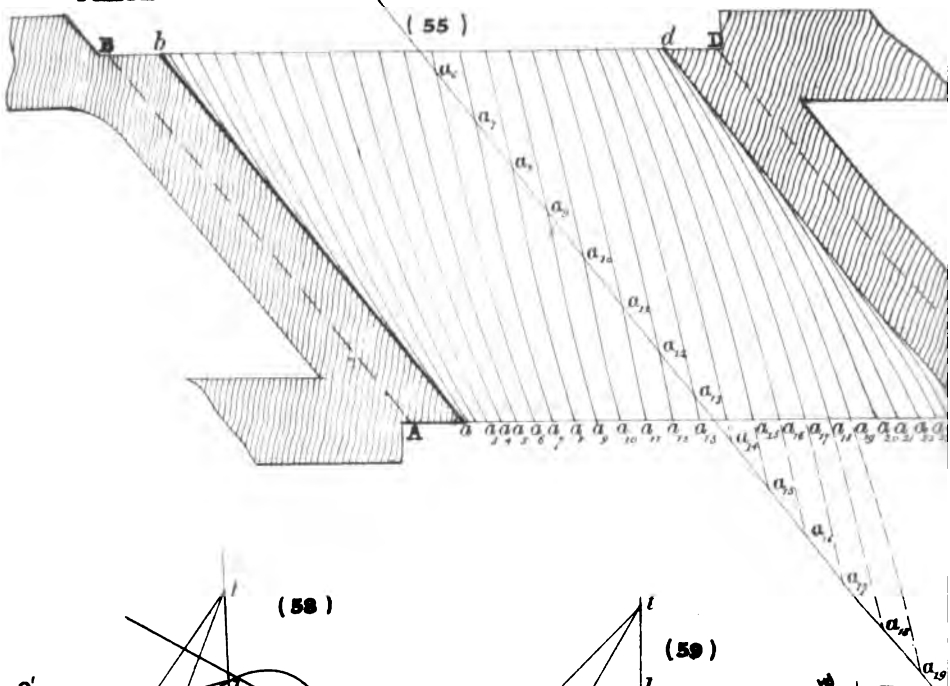


PLATE 10

(55)



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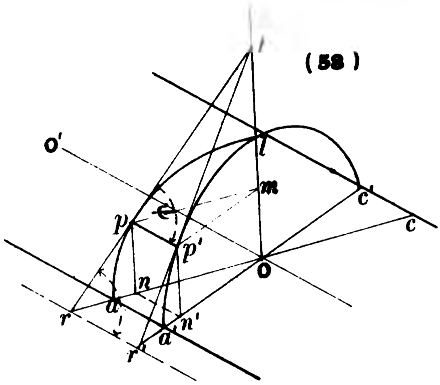
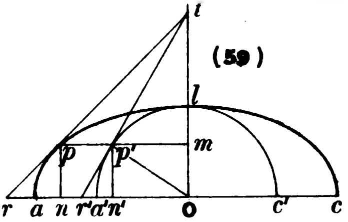
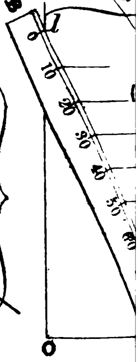
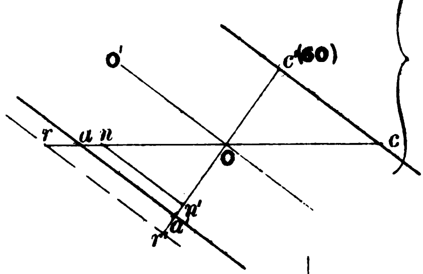


(59)

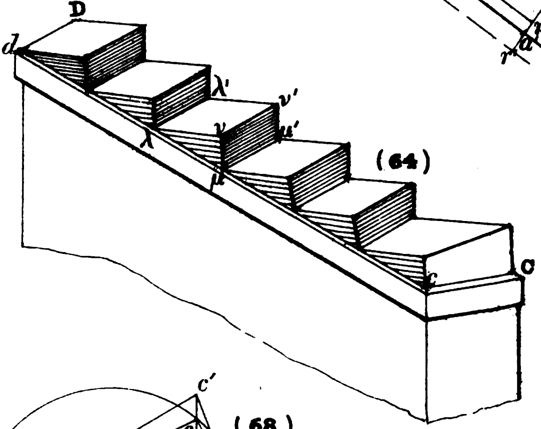




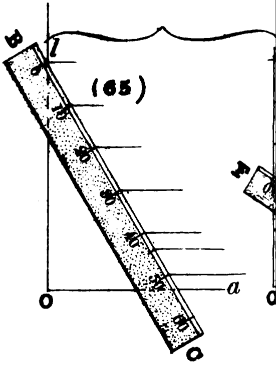
(60)



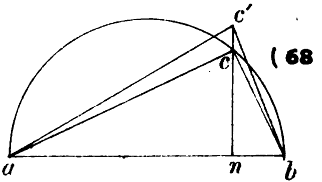
(64)



(65)



(68)



F. F.

(69)

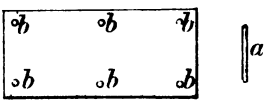
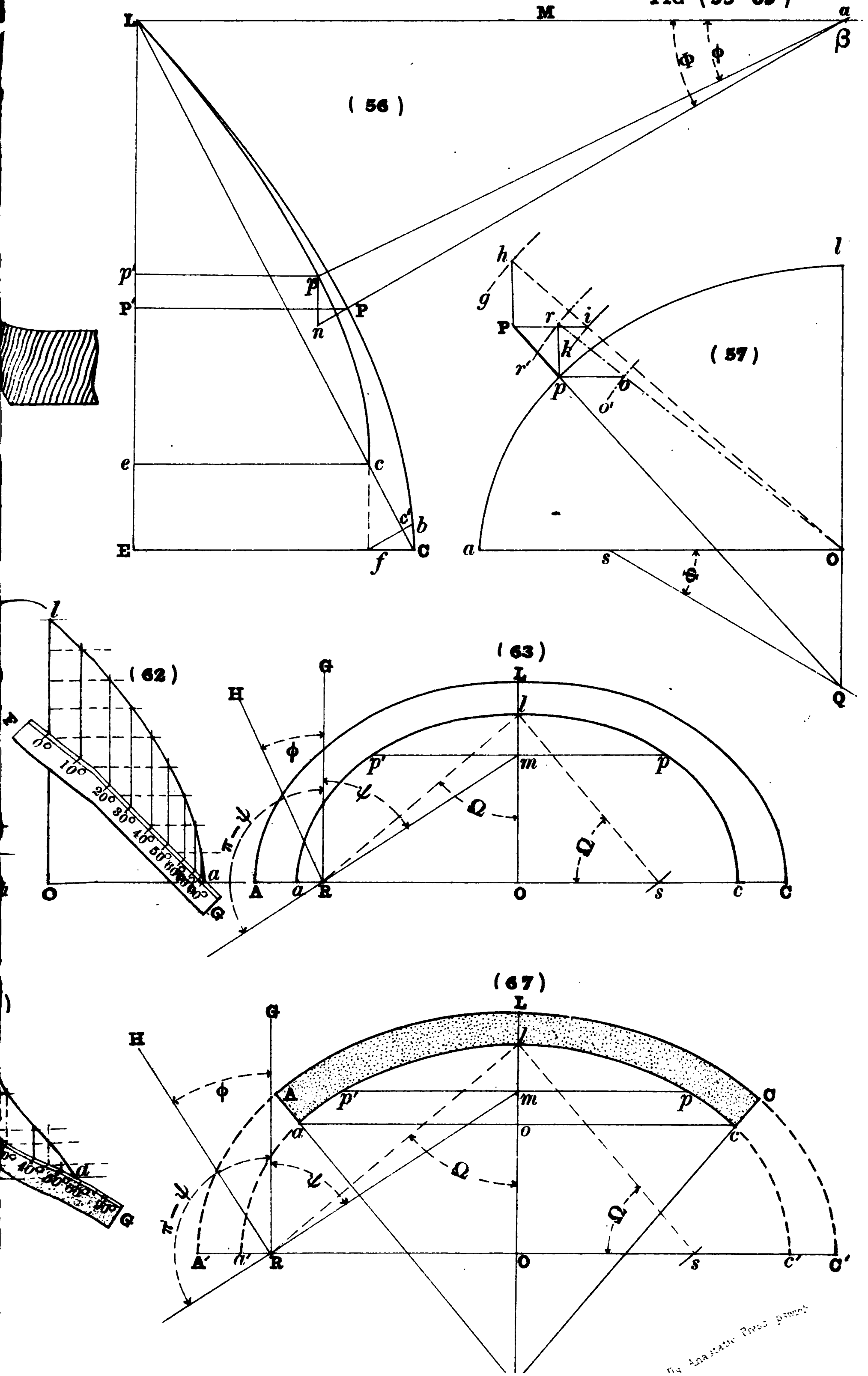


FIG (55–69)



By Anastatic Press

PLATE 11

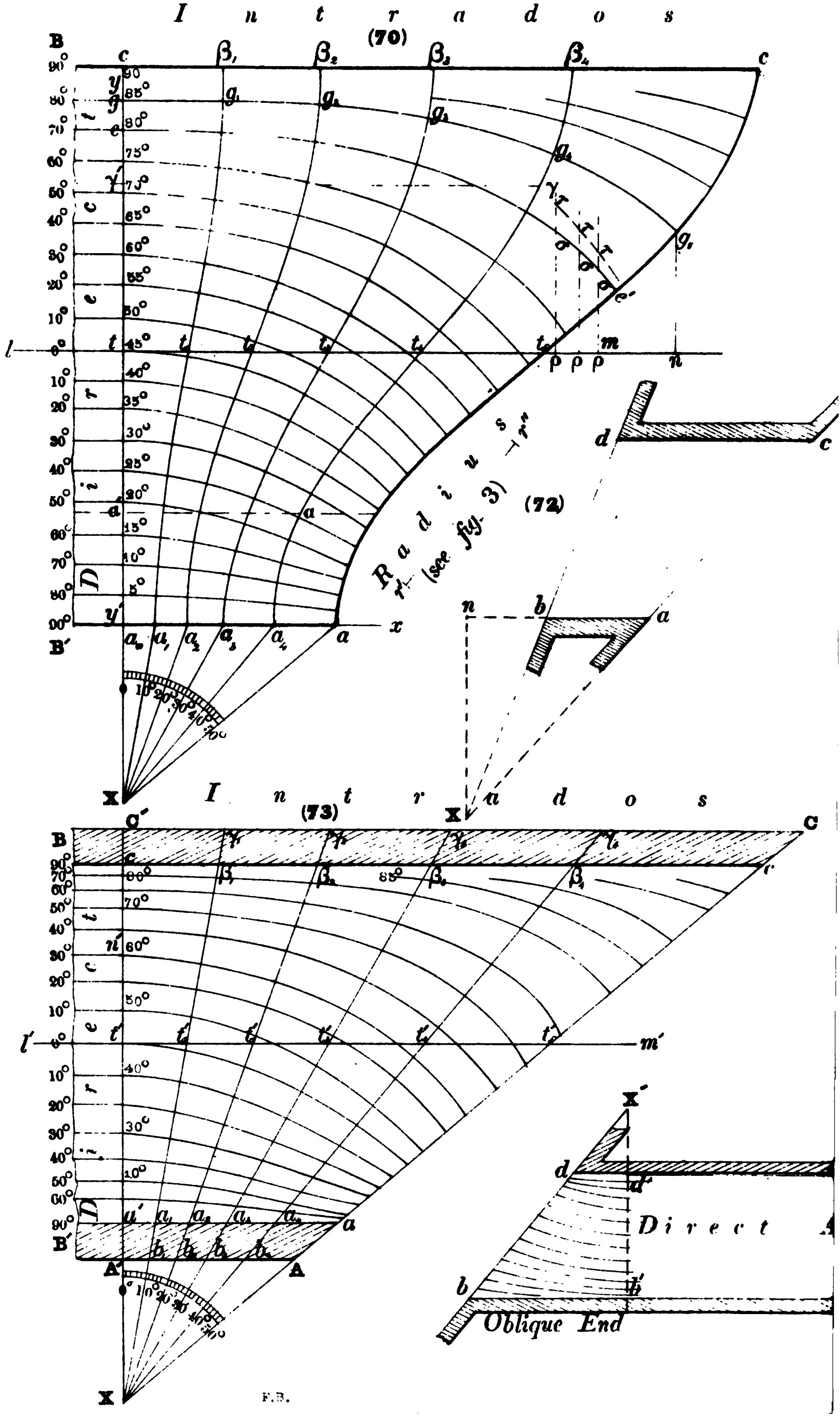
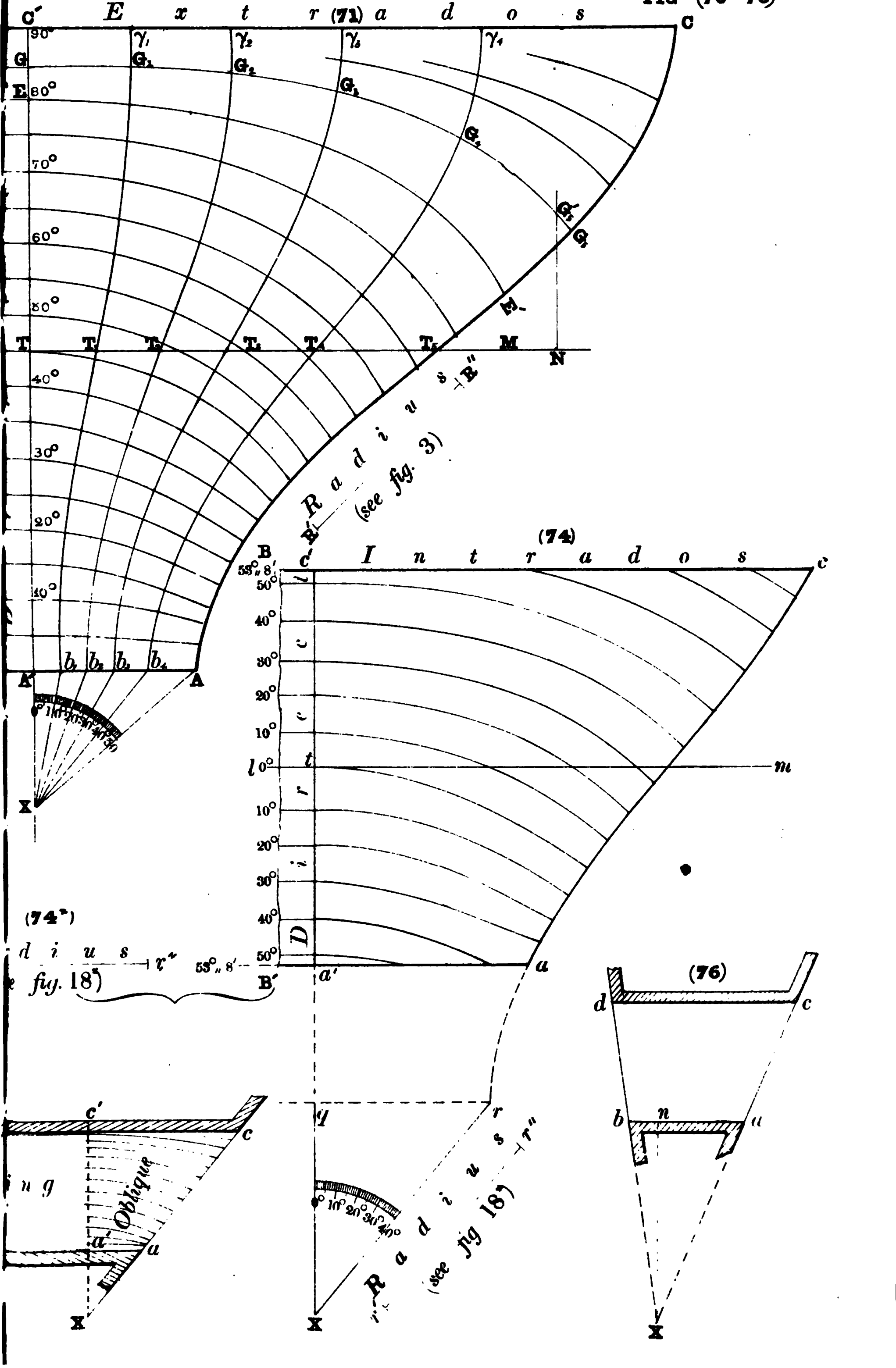

I n t r a d o s
(70)
B
c
β,
β₂
β₃
β₄
c
90° 80° 70° 60° 50° 40° 30° 20° 10° 0° 10° 20° 30° 40° 50° 60° 70° 80° 90°
90 85° 80° 75° 70° 65° 60° 55° 50° 45° 40° 35° 30° 25° 20° 15° 10° 5°
y
g
e
γ'
g,
g₂
g₃
g₄
γ
r
ρ
g,
e'
l
t
t,
t₂
t₃
t₄
t₅
m
ρ ρ ρ
n
D i r e c t
d
a
y'
B'
a₀ a, a₂ a₃ a₄ a
x
10° 20° 30° 40° 50°
X
R a d i u s
r'' r'
(see fig. 3)
(72)
d
c
n
b
a
I n t r a d o s
(73)
X
C'
C
c
c
80° 85° 70° 60° 50° 40°
β, β₂ β₃ β₄
n'
l'
t'
t,
t₂
t₃
t₄
t₅
m'
u'
a, a₂ a₃ a₄
a
b b₂ b₃ b₄
A'
A
X'
d
d'
D i r e c t
b
b'
Oblique End
X
F.B.

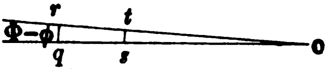
FIG (70--76)



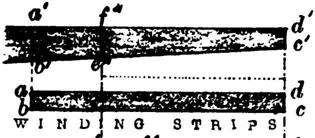
(71)
C′ E x t r a d o s C
90° γ1 γ2 γ3 γ4
G G1 G2 G3 G4 G5 G′5
E 80°
70°
60°
50°
T T T2 T3 T4 T5 M N
E-
40°
30°
20°
10°
R a d i u s R″
(see fig. 3)
R′
A′ b1 b2 b3 b4 A
0° 10° 20° 30° 40° 50°
X
(74)
B
c′ I n t r a d o s c
53°„ 8′
50° 40° 30° 20° 10°
l 0° t m
10° 20° 30° 40° 50°
D i r e c t
a′ a
B′
53°„ 8′
(74*)
R a d i u s r″
see fig. 18″
c′ c
Oblique
n g
a′ a
X
q r
0° 10° 20° 30° 40°
R a d i u s r″
(see fig 18″)
X
(76)
d c
b n a
X

PLATE 12

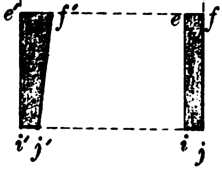
(77)



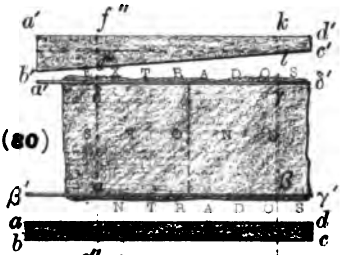
(78)



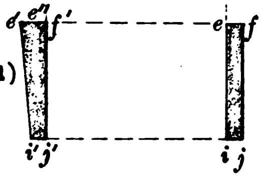
(79)



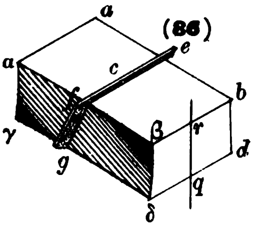
(80)



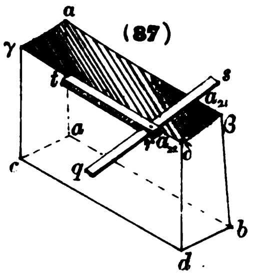
(81)

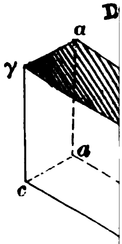


(86)

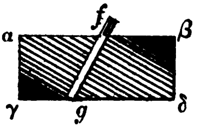


(87)

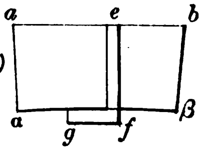




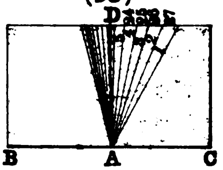
(91)



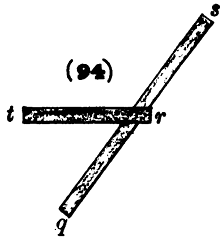
(92)

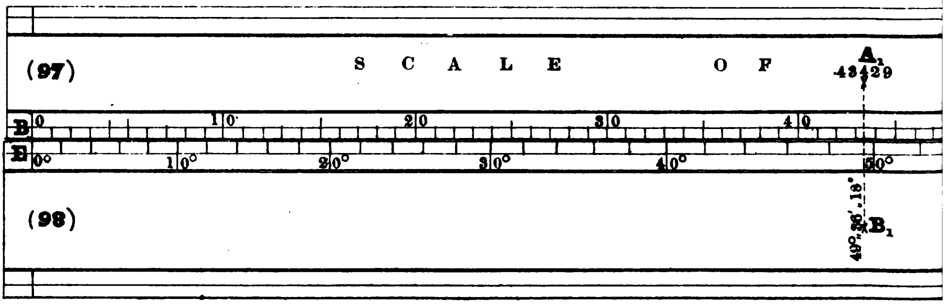


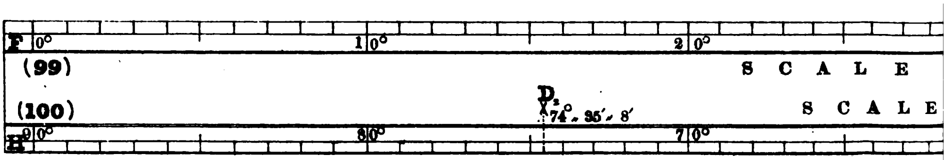
(93)



(94)



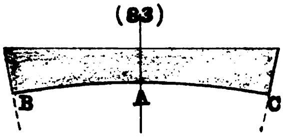




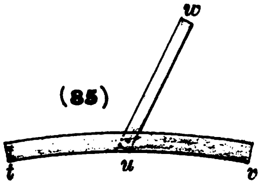
P.B.

FIG (77—100)

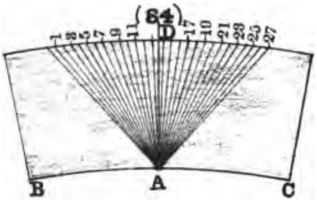
(83)



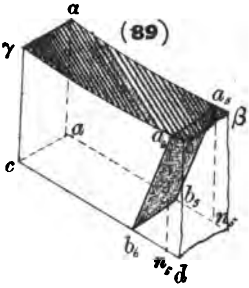
(85)



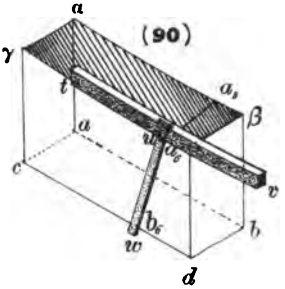
(84)



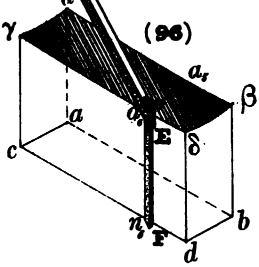
(89)

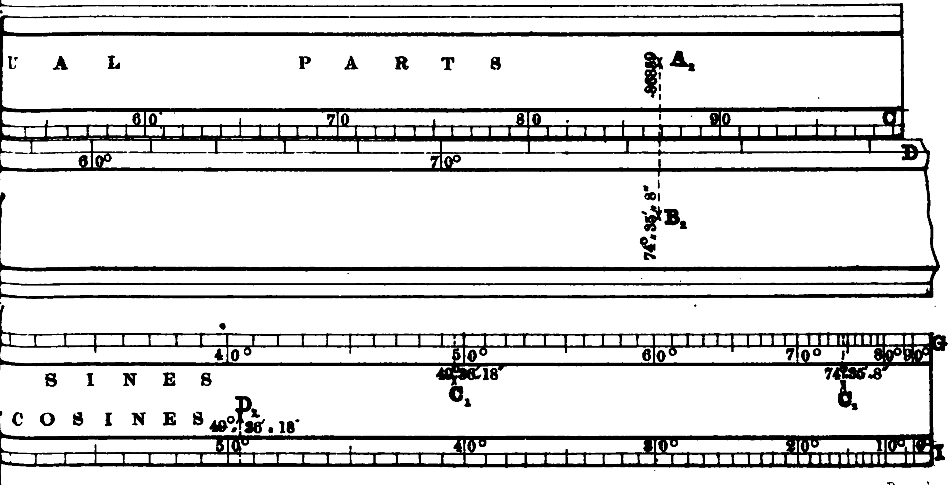


(90)



(96)





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